Multifrontal Sparse QR Factorization on GPU

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Outline

- ▶ **Introduction**
  - Multifrontal sparse QR
  - GPU considerations
- **GPU-based multifrontal sparse QR**
  - Memory traffic
  - Uberkernel
  - Front scheduling
  - Pipelined dense QR factorization

Multifrontal Sparse QR

- **Why QR?**
  - Wide Applicability
  - Numerically Stable
  - High ratio of flops to memory accesses
- **Why Multifrontal?**
  - CAQR (2008, Demmel et al)
  - No data dependence during child assembly
- **Why GPU?**
  - Teraflops of peak theoretical performance
  - Low amortized cost per flop (watts, dollars)

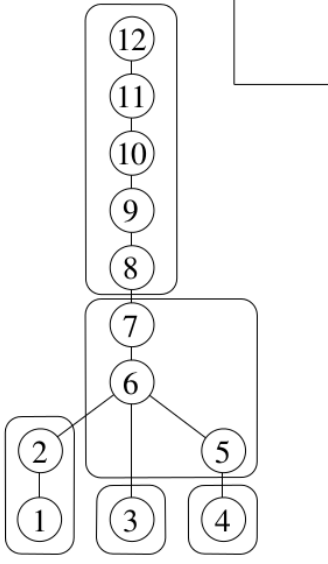
Multifrontal Sparse QR

the matrix A

	1	2	3	4	5	6	7	8	9	10	11	12
1	x	x				.		x			.	
2	x	.				x		.			x	
3	x	x				.		x			x	
4		x				x		.			x	
5		x				x		.			x	
6		x				.		x			.	
7			x			x	x	.				x
8			x			.	.	.				x
9			x			x	.	x				.
10			x			.	x	x				.
11			x			.	x	.				.
12				x	.		.		.		x	
13				x	x		x		x		.	
14				x	x		.		x		x	
15				x	x		.		.		x	
16					x	.	x	x	.		x	x
17					x	x	.	.	.		.	x
18					x	x	.	x	x		x	.
19					.	x	x	.	x		.	x
20					.	x	.	x	.		.	x
21					.	.	x	.	.		x	x
22					.	.	x	x	x		x	x
23								x	x	x	x	x

the QR factor R

	1	2	3	4	5	6	7	8	9	10	11	12
1	r	r				r		r			r	
2		r				r		r			r	
3			r			r	r	r				r
4				r	r		r		r		r	
5					r	r	r	r	r		r	r
6						r	r	r	r		r	r
7							r	r	r		r	r
8								r	r	r	r	r
9									r	r	r	r
10										r	r	r
11											r	r
12												r



the column elimination tree

Multifrontal Sparse QR

child front 1

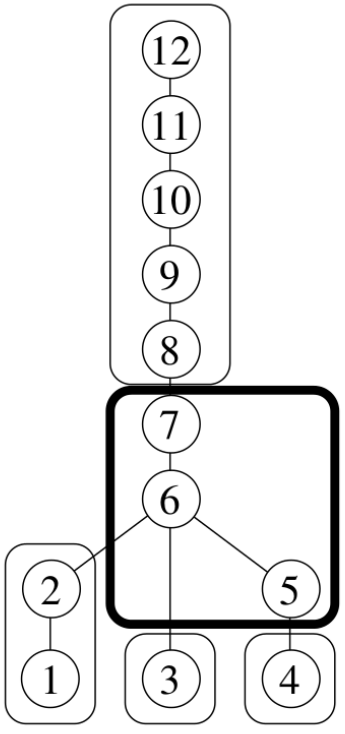
	6	8	11
3	c_1	c_1	c_1
4		c_1	c_1
5			c_1

child 3

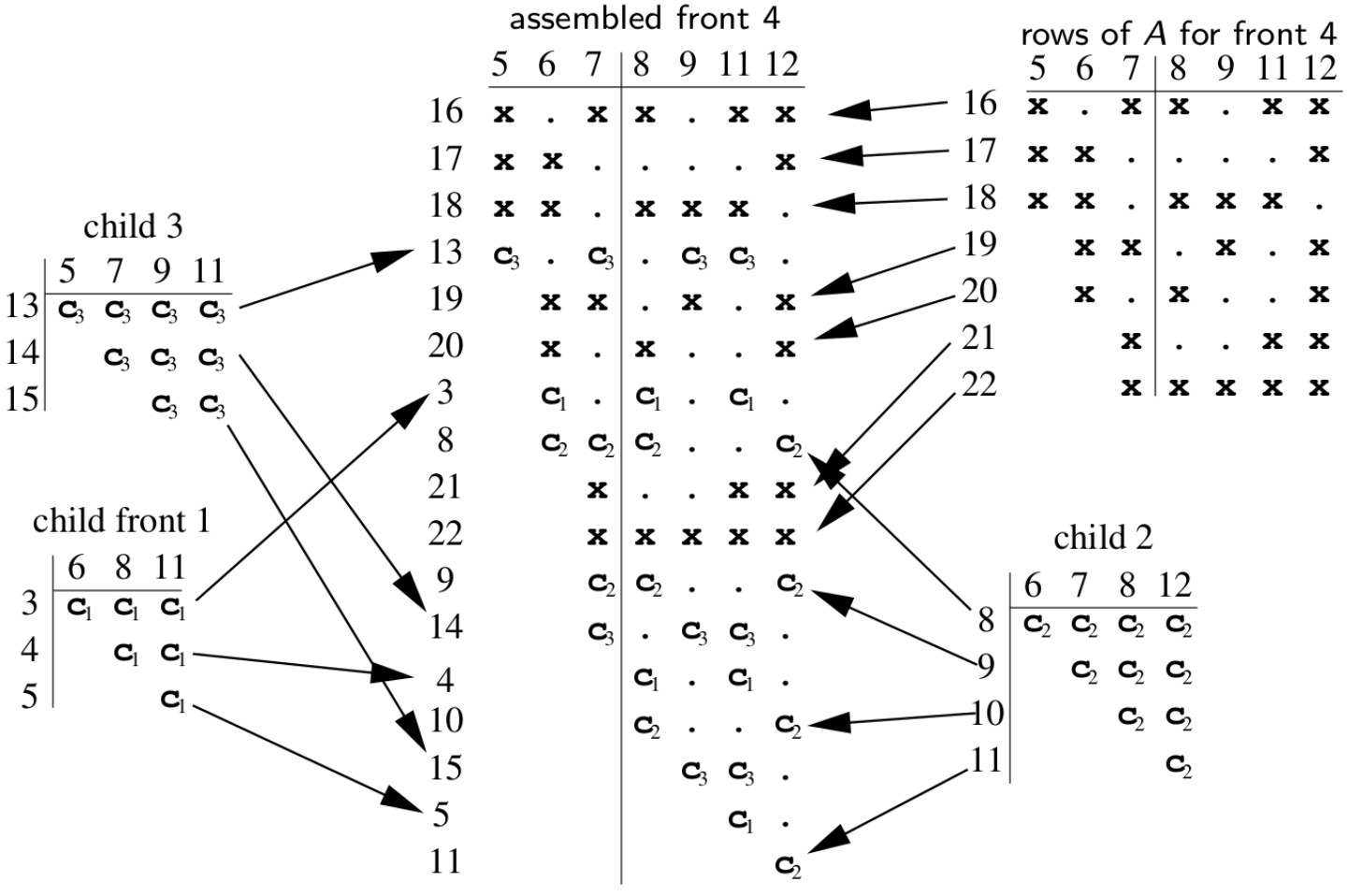
	5	7	9	11
13	c_3	c_3	c_3	c_3
14		c_3	c_3	c_3
15			c_3	c_3

child 2

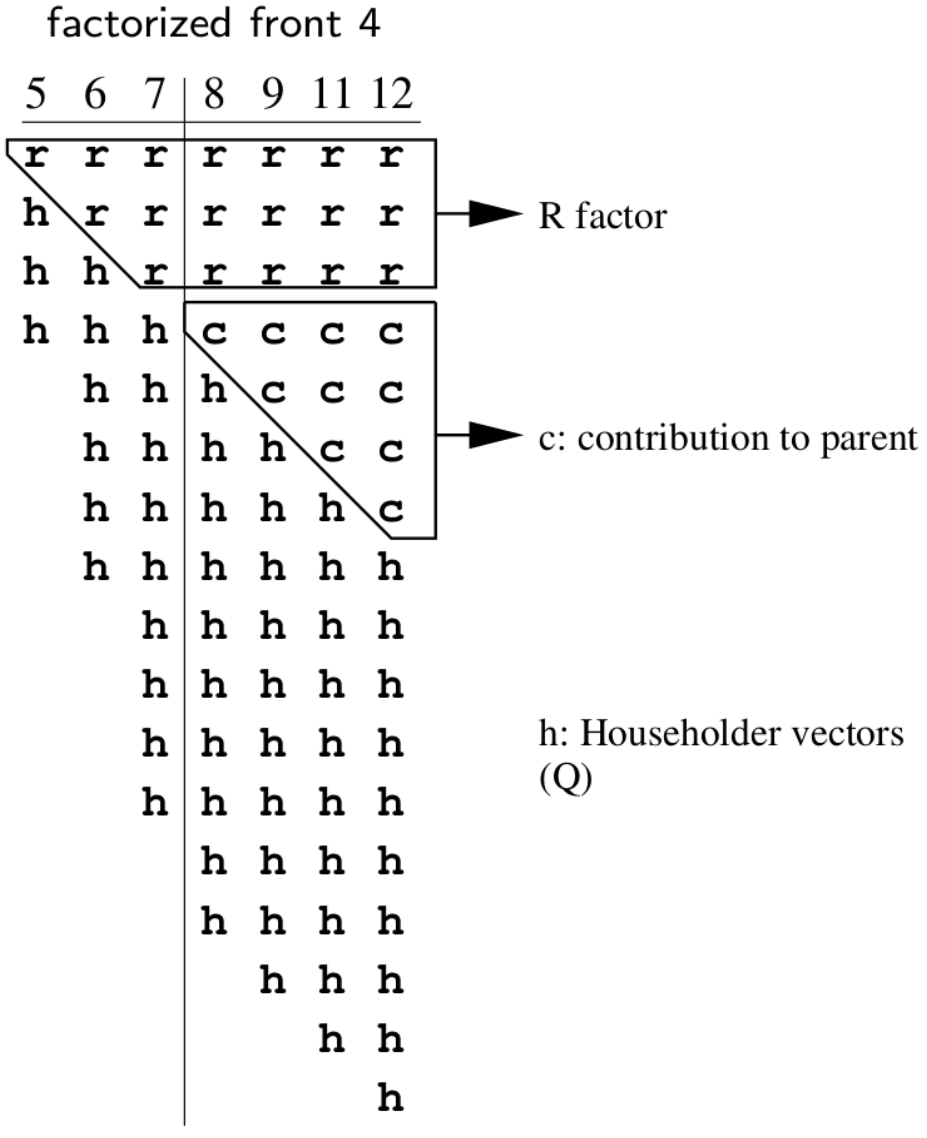
	6	7	8	12
8	c_2	c_2	c_2	c_2
9		c_2	c_2	c_2
10			c_2	c_2
11				c_2



Multifrontal Sparse QR



Multifrontal Sparse QR



GPU Considerations

- **Memory limitations (availability, bandwidth)**
  - Maintain high ratio of computation to memory transfers
  - Use streams to mitigate transfer costs
  - Find opportunities to exploit locality & reuse memory
  - Use memory coalescing, "shared" memory, and register tricks
  - Avoid excessive cudaMalloc calls
- **Synchronization**
  - GPUs are great for data parallelism, bad for task parallelism
  - Ensuring memory consistency damages performance
  - No reliable inter-block synchronization primitives..yet

Outline

- **Introduction**
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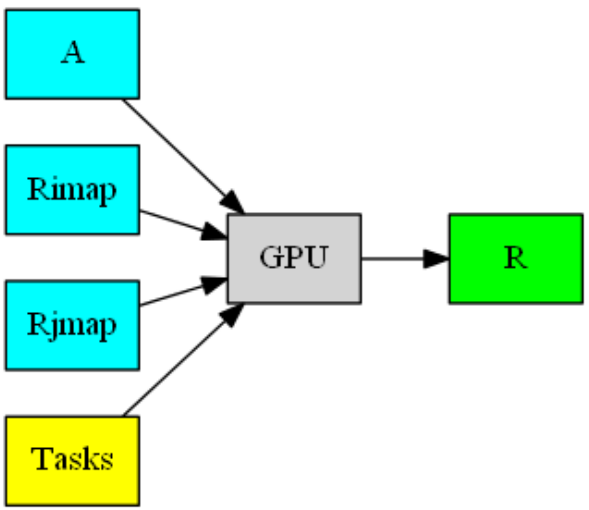
  - GPU considerations
- ▶ **GPU-based multifrontal sparse QR**
  - Memory traffic
  - Uberkernel
  - Front scheduling
  - Pipelined dense QR factorization

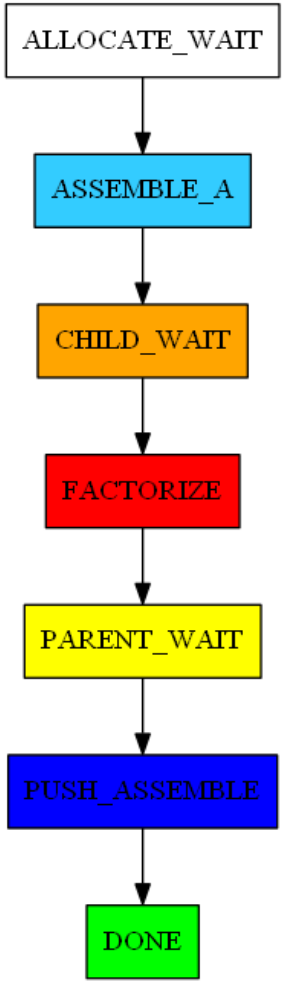
- **No superfluous memory traffic**
  - Data is transfered asynchronously, sometimes surgically
    - We can accomodate lazy JIT transfers if needed
  - Every front spends its entire life on the GPU
    - It's allocated once any of its children are finished factorizing
    - It assembles its contribution block directly into its parent
    - Its memory is recovered once its rows of R are off the GPU



GPU-based Multifrontal Sparse QR

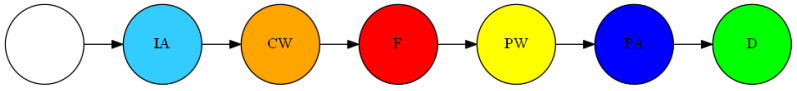
- **The uberkernel pattern allows us to process different types of tasks in the same kernel launch**
  - Initial Assembly (Input Problem to Front)
  - Householder Annihilate
  - Block Householder Apply
  - Pipelined Block Householder Apply & Annihilate
  - Push Assembly (Child to Parent)
- **Treat kernel launches as barrier syncs, and do as much work as possible per launch.**
  - Oversubscribe the GPU with work
  - Use events and streams to:
    - Launch the kernels asynchronously and build the next body of work while the GPU is busy
    - Ship the next set of tasks asynchronously

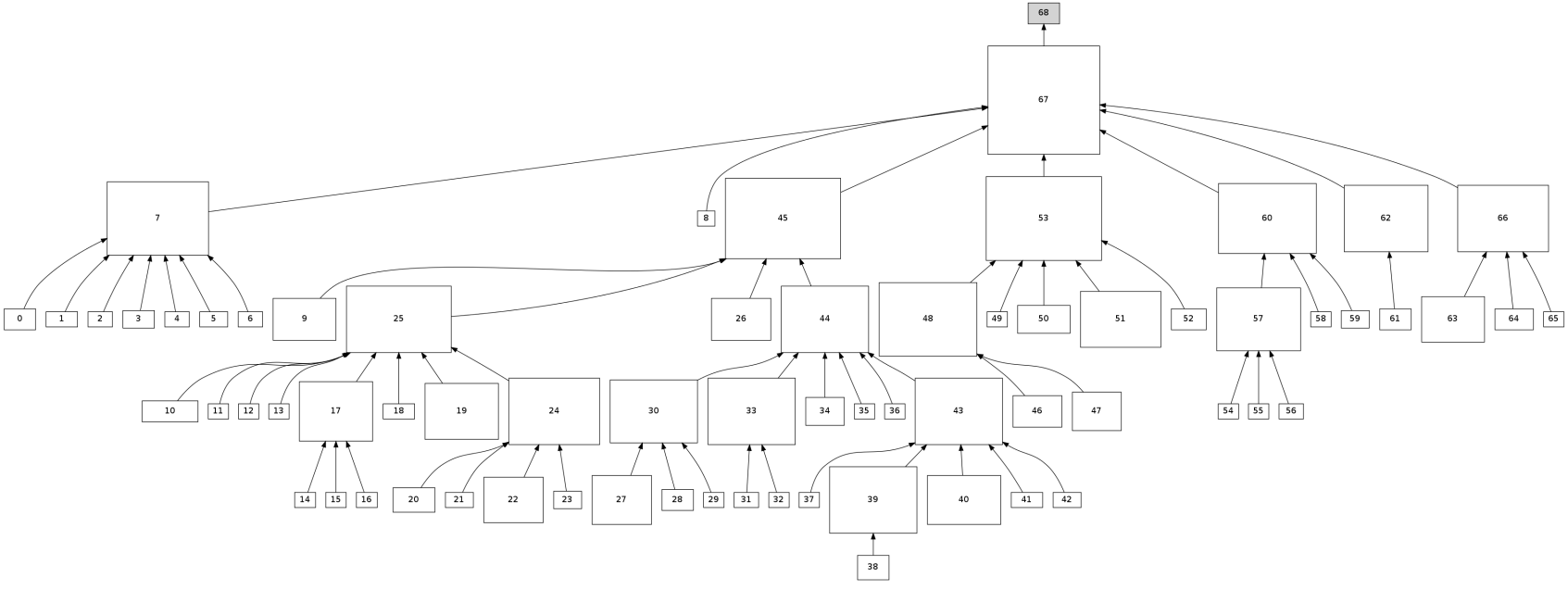
- **FSM-driven dynamic scheduling**



- **What does this look like?**
  - "The most effective debugging tool is still careful thought, coupled with judiciously placed print statements."
    - Brian Kernighan, "Unix for Beginners" (1979)

Front Scheduling





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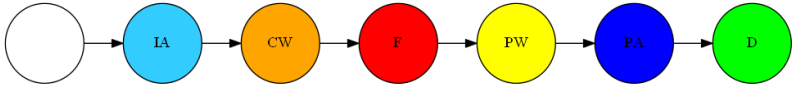

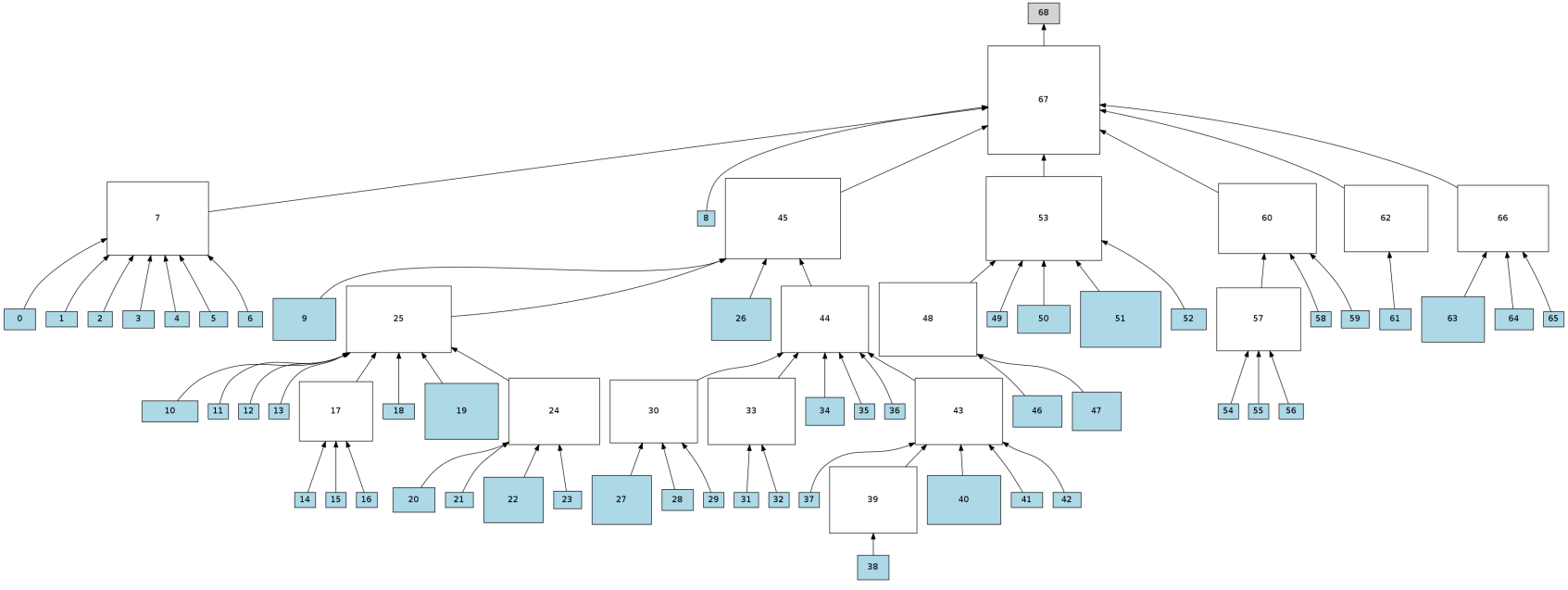

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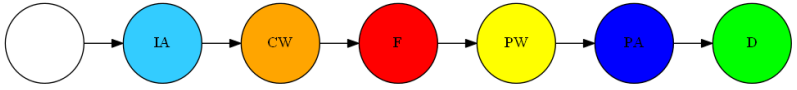

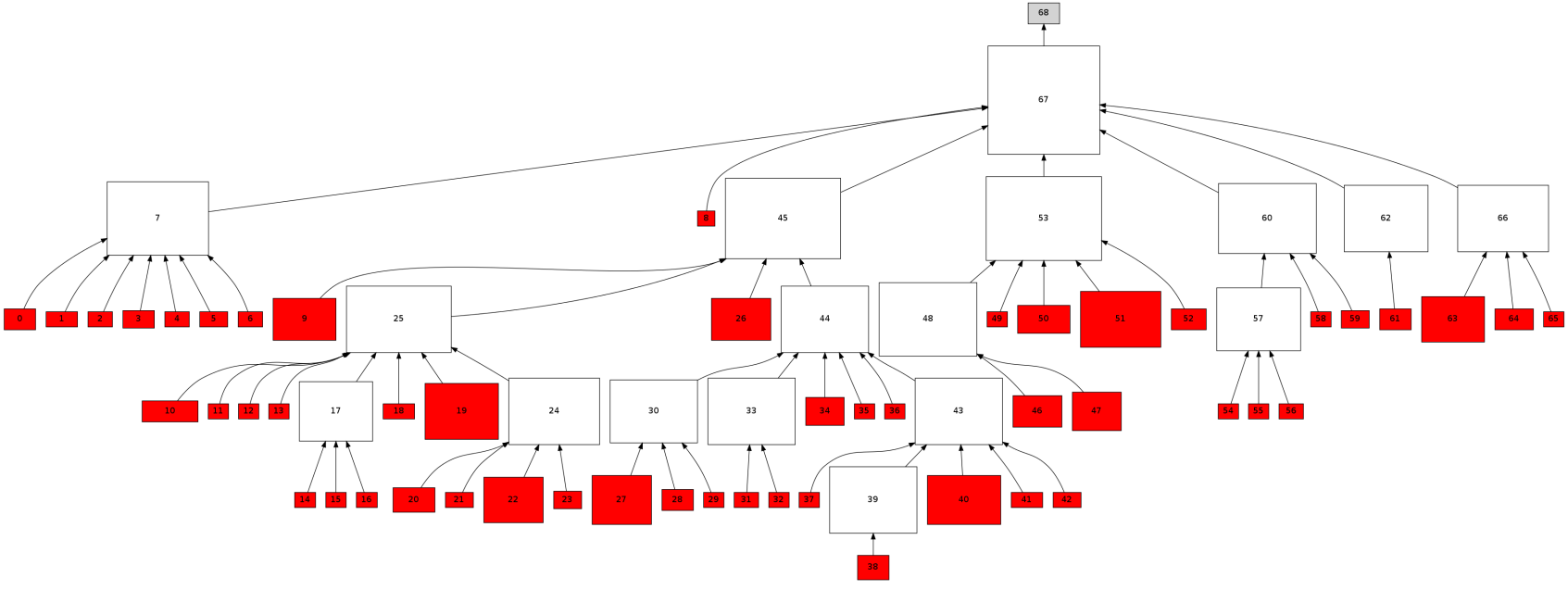

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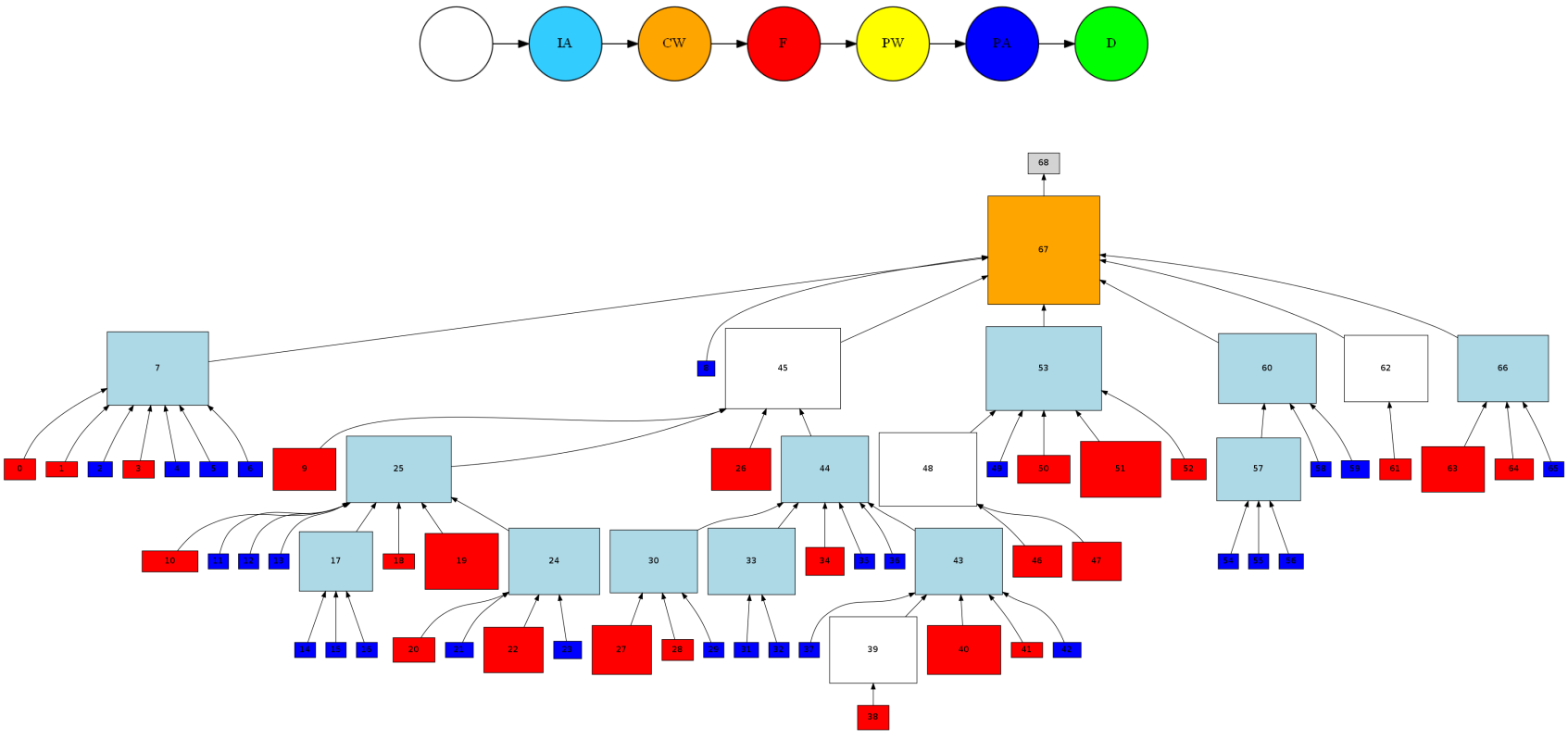

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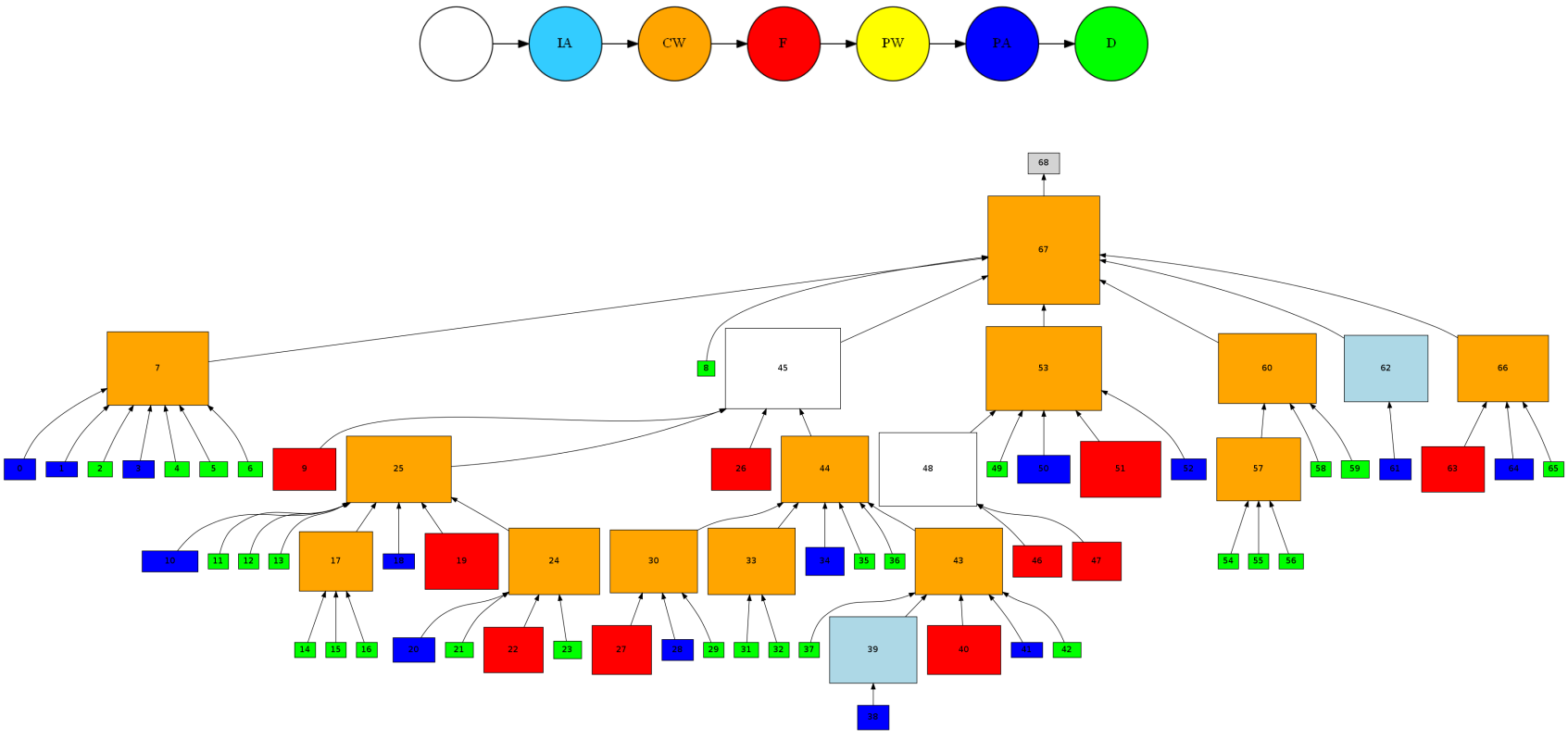

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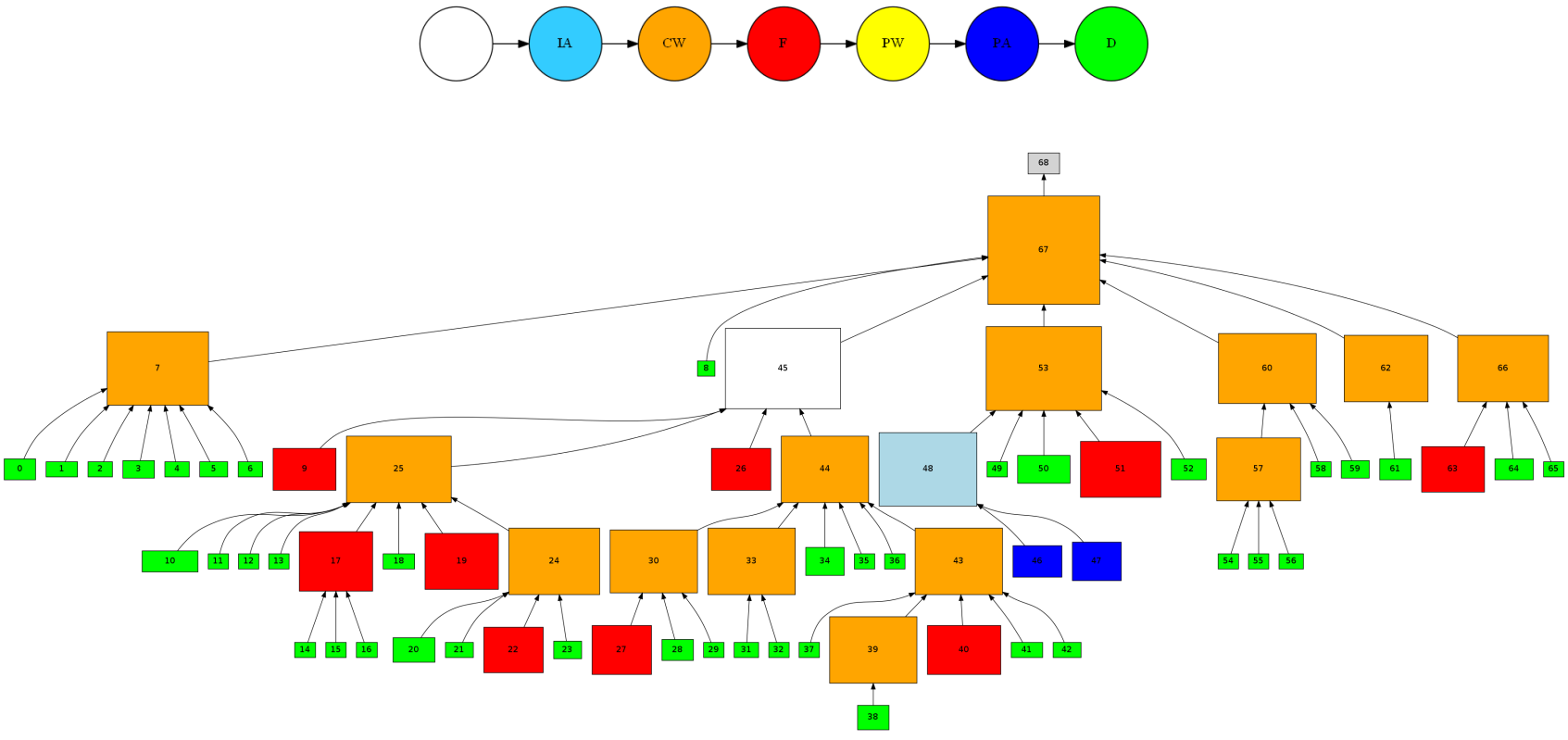

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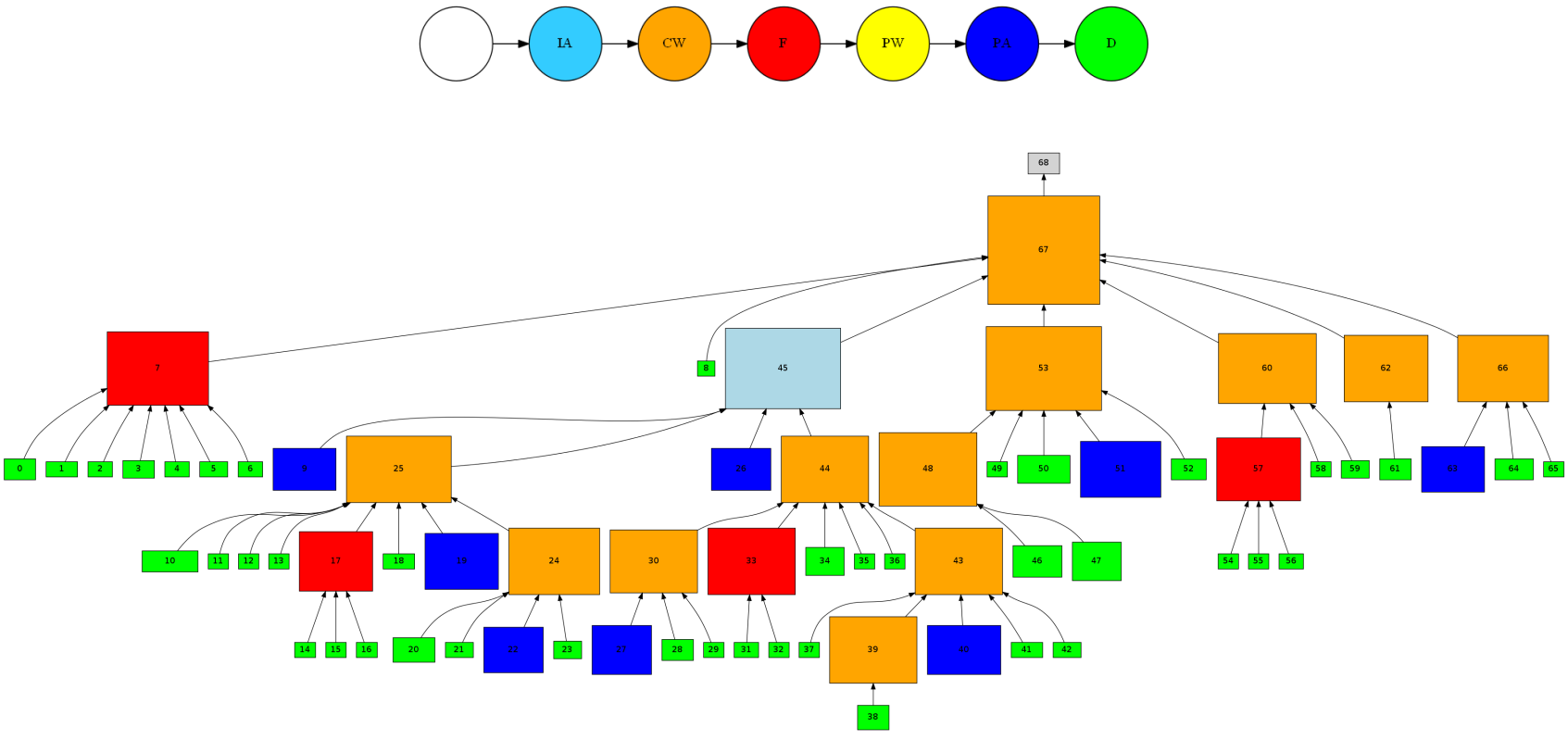

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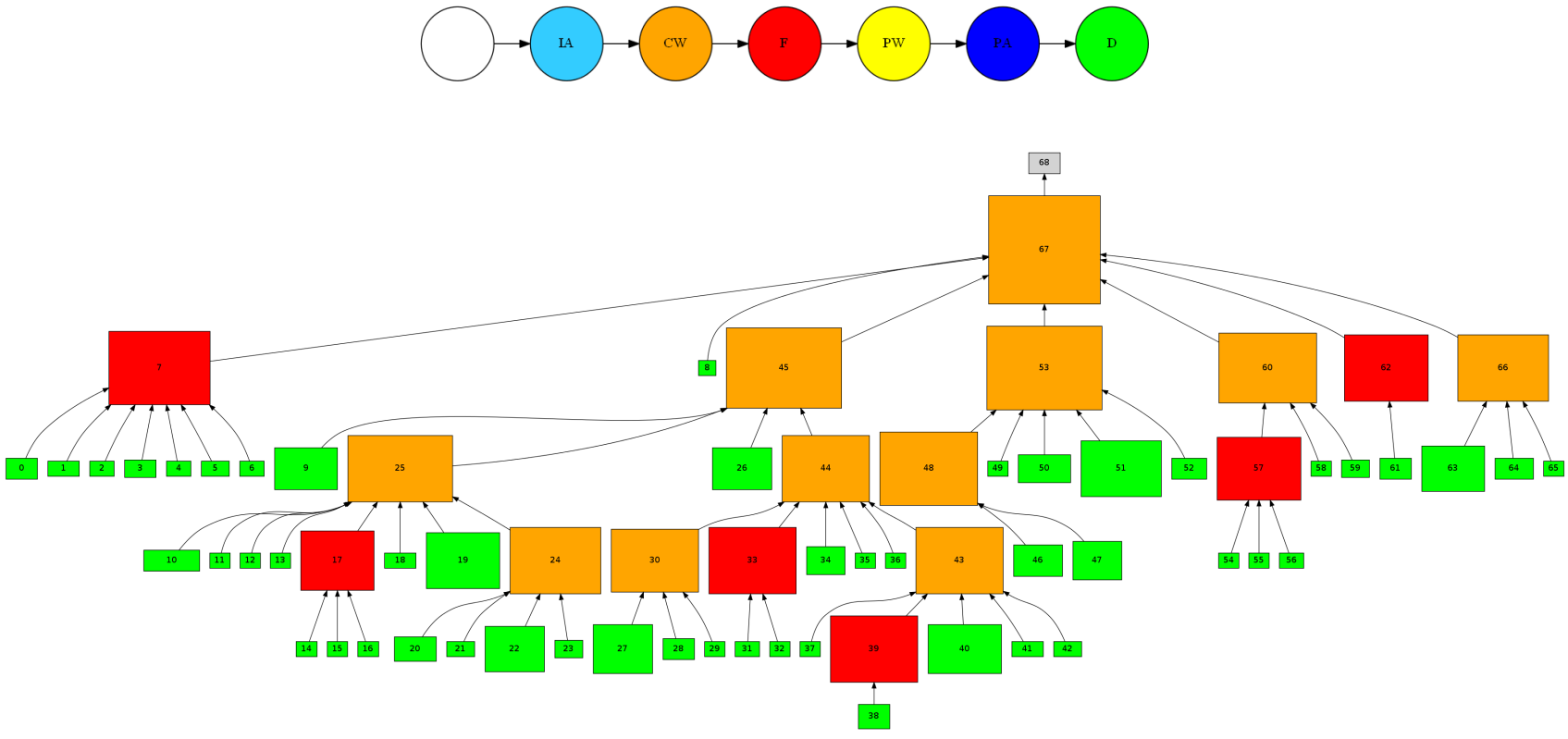

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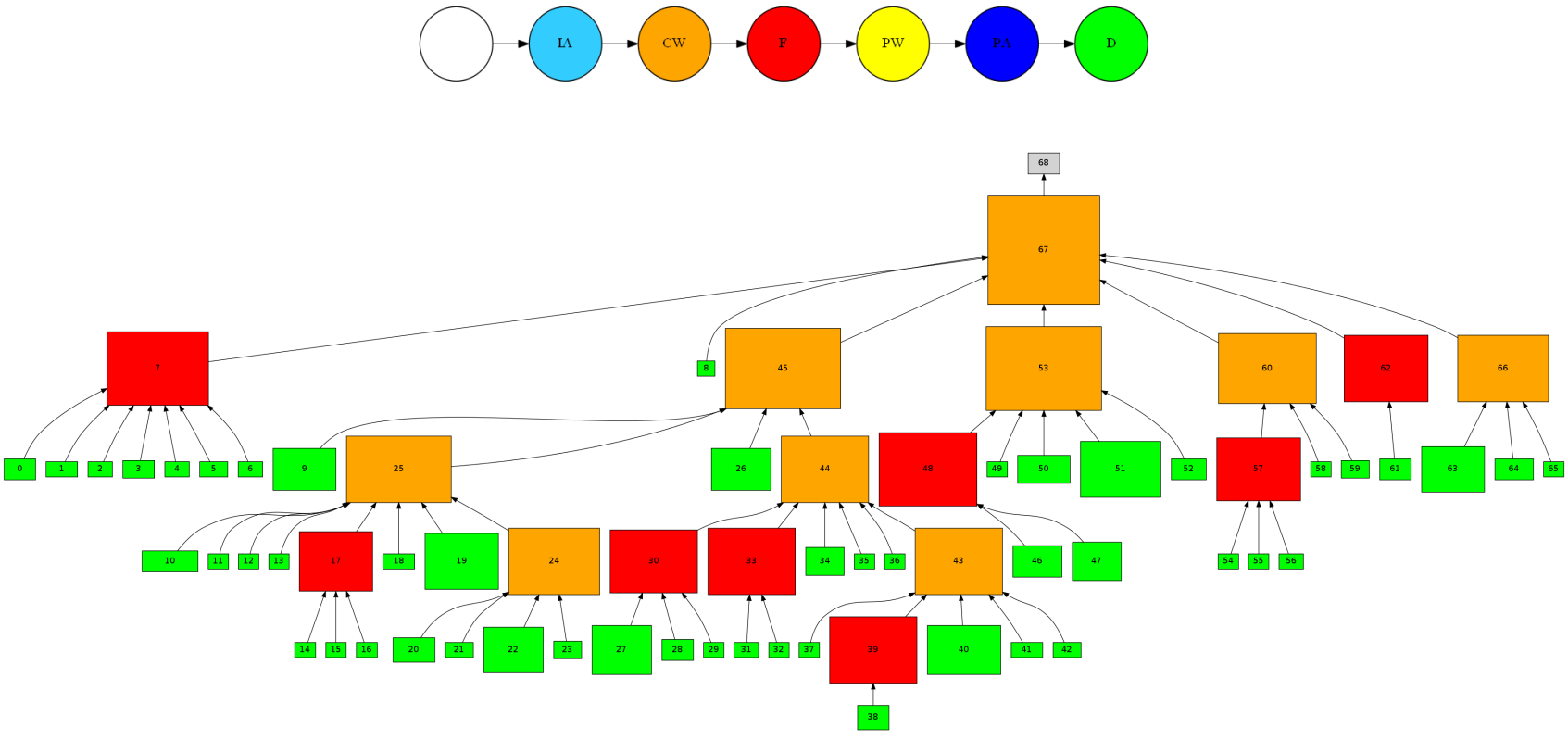

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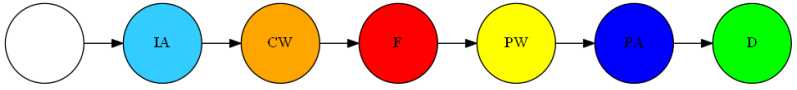

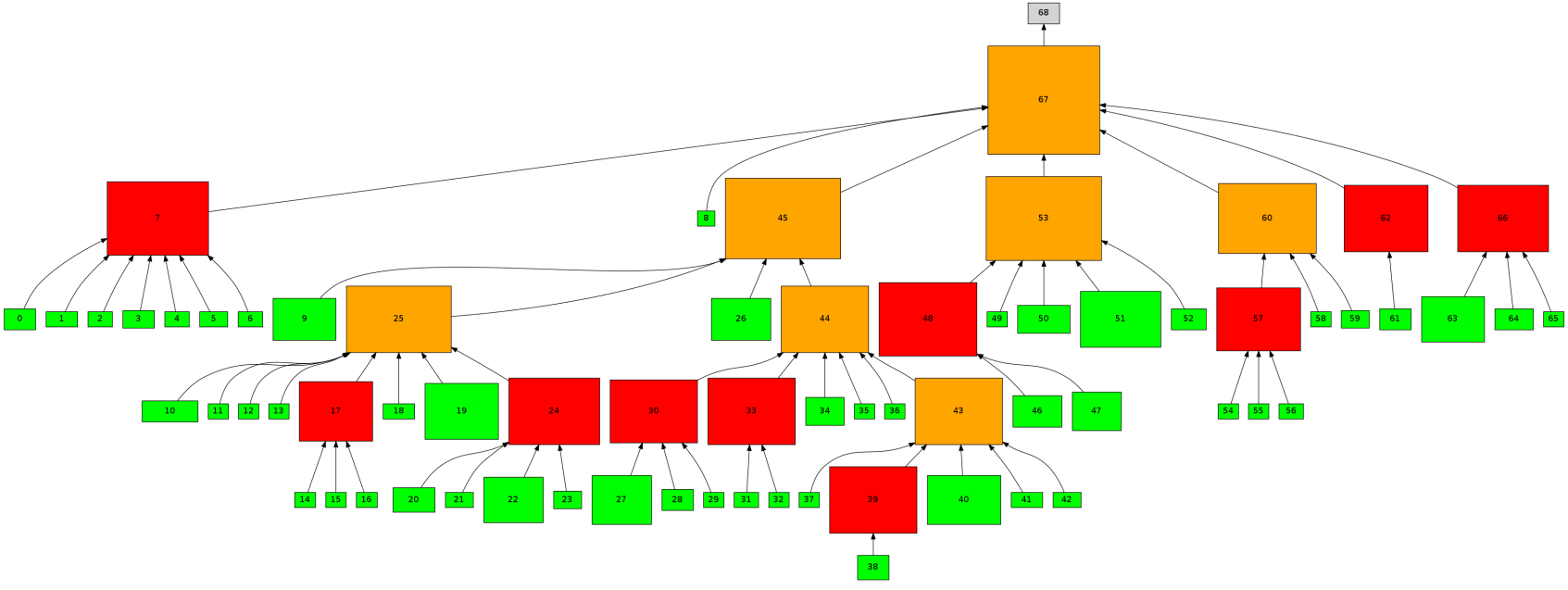

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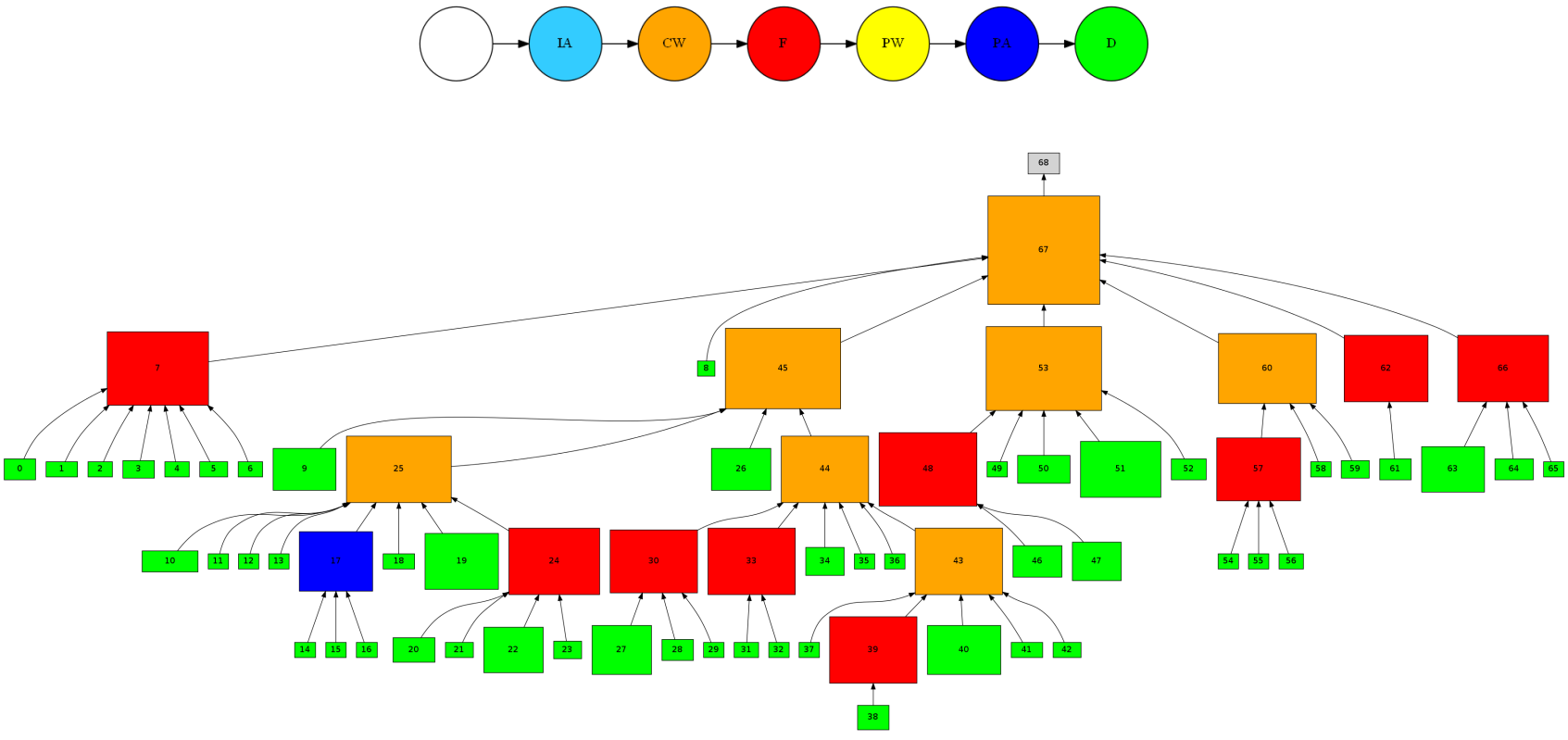

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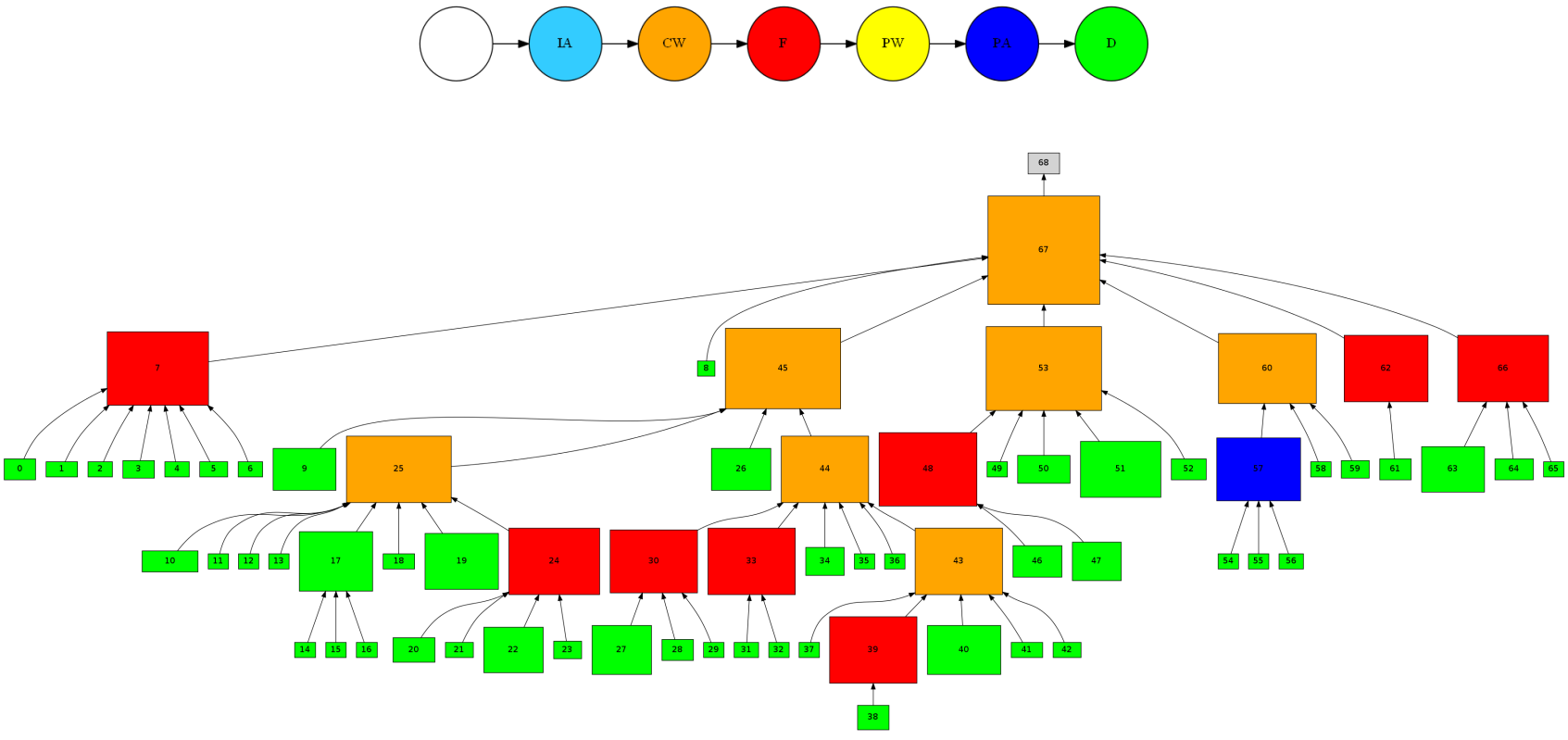

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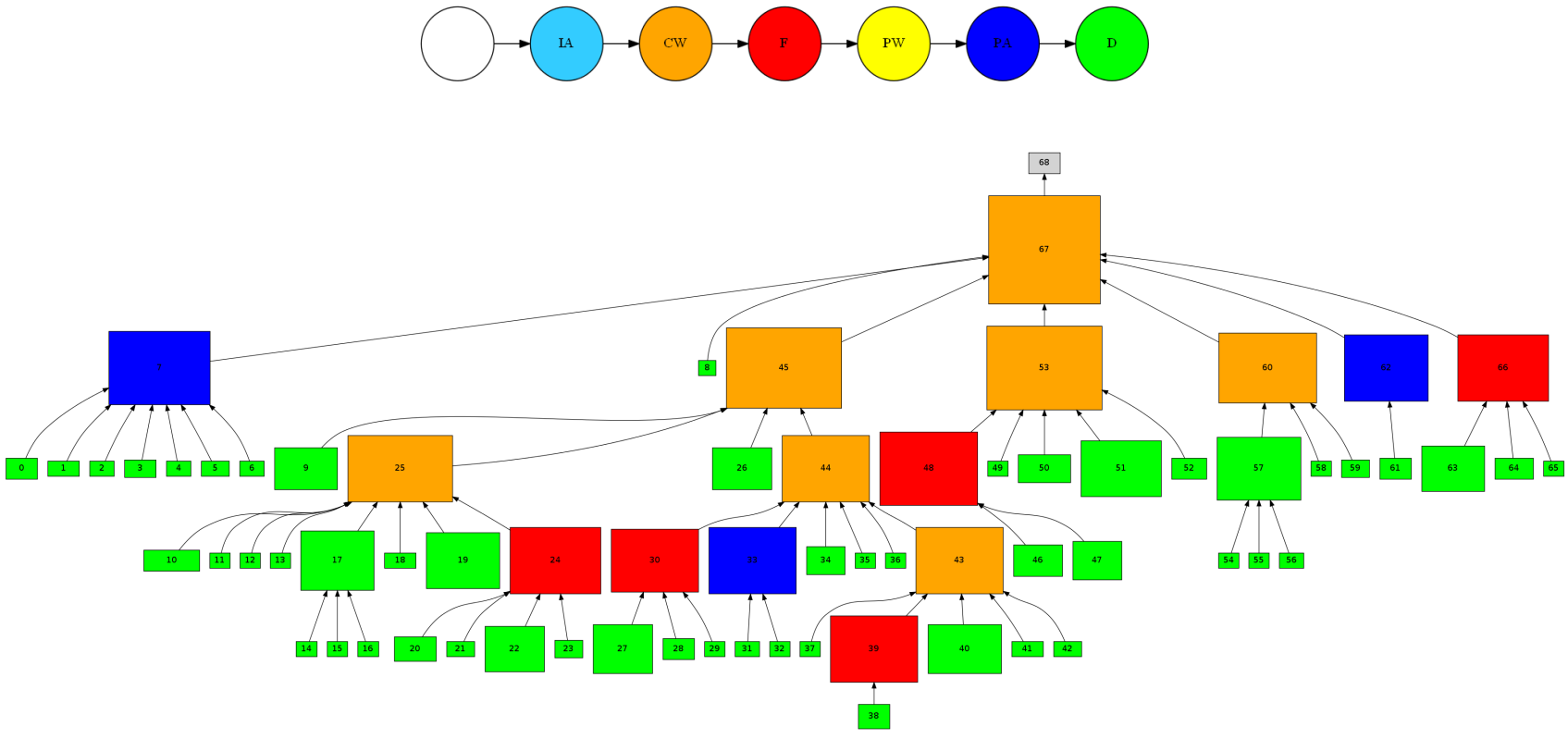

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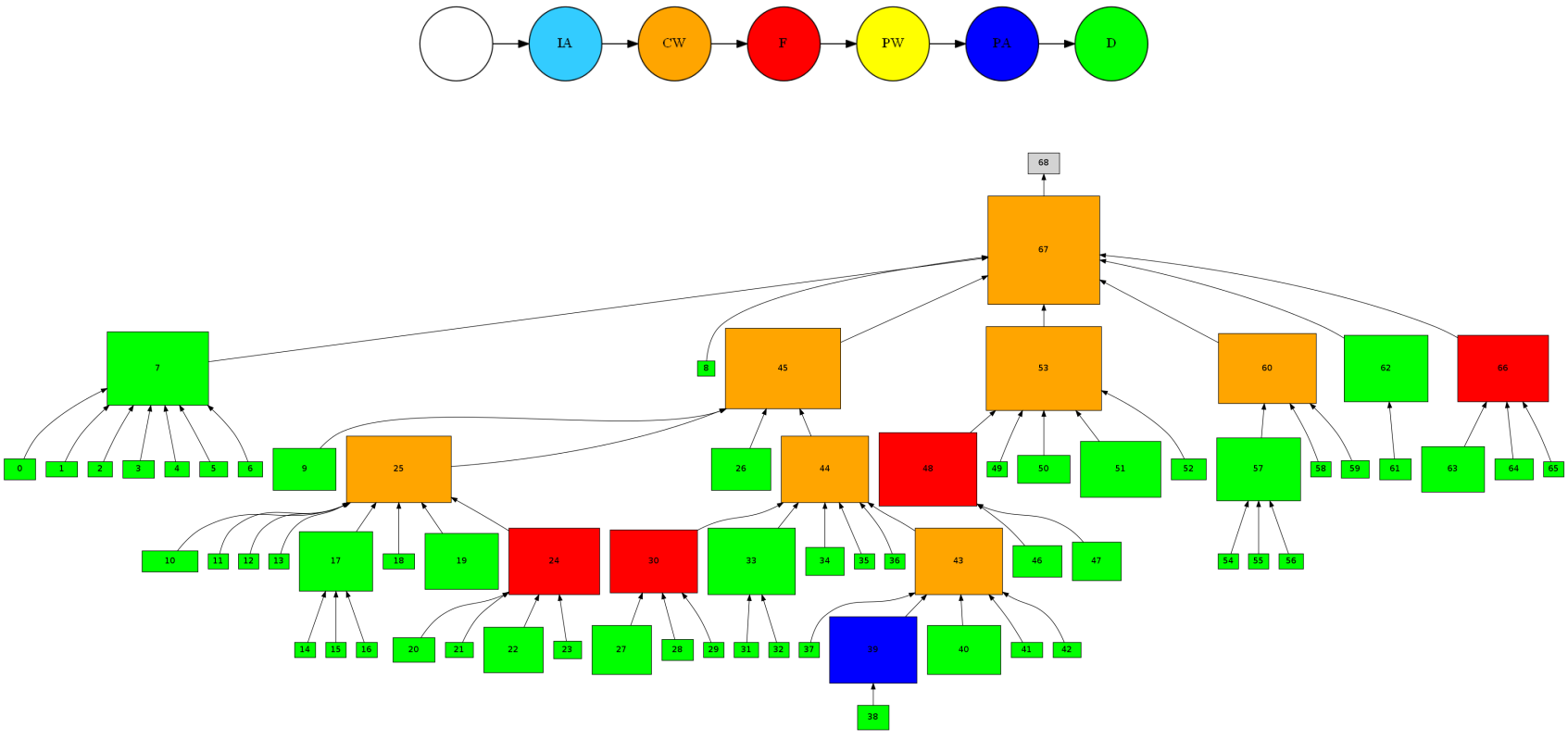

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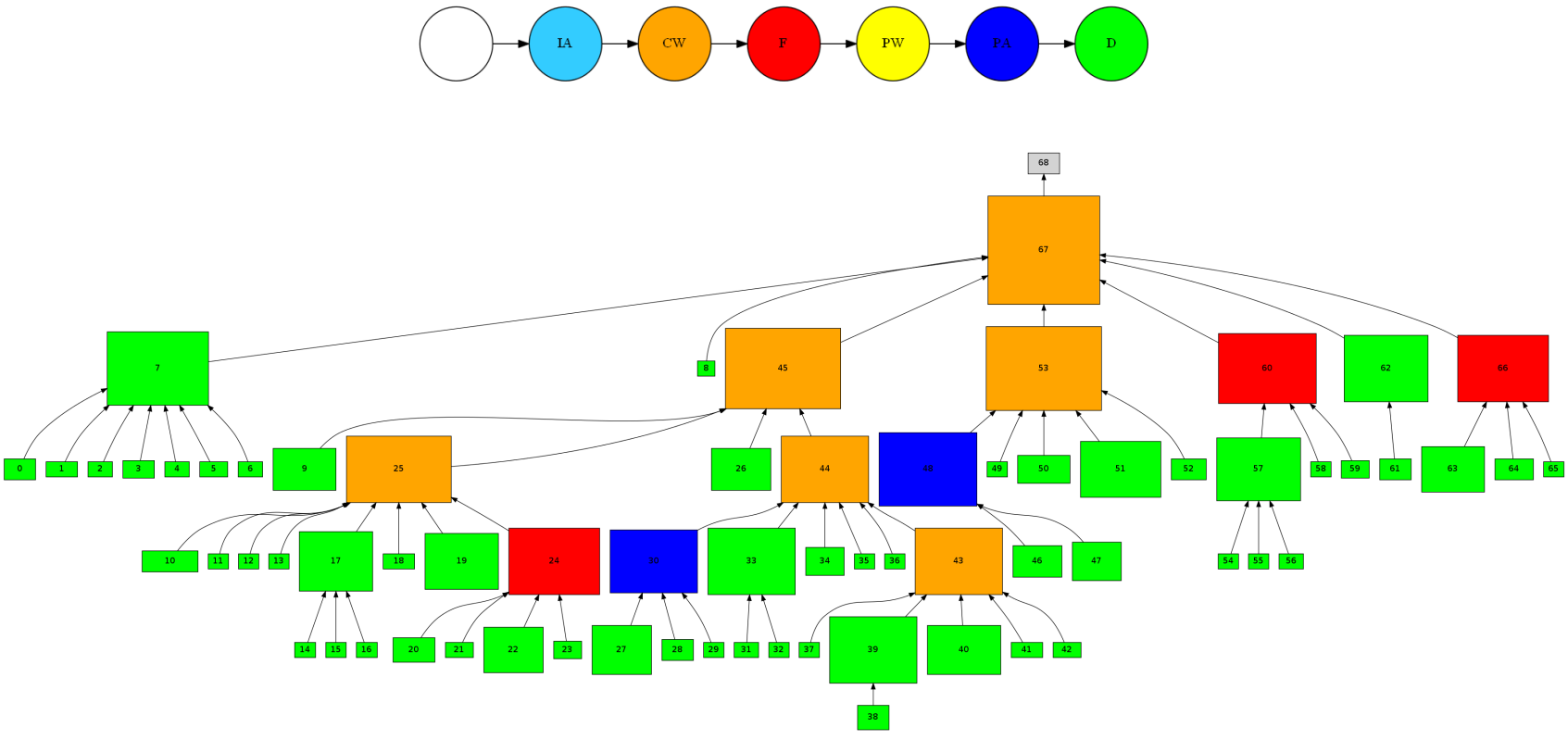

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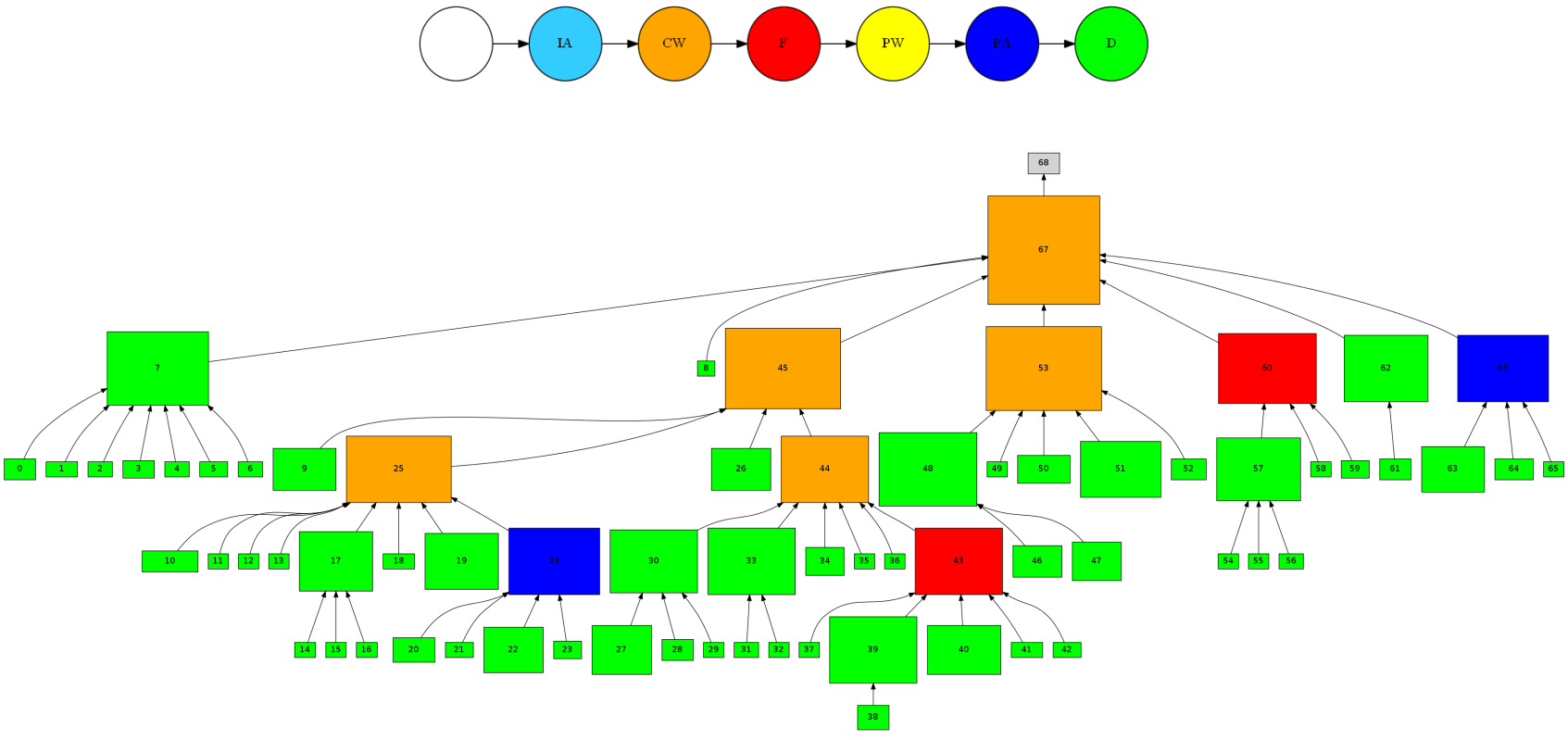

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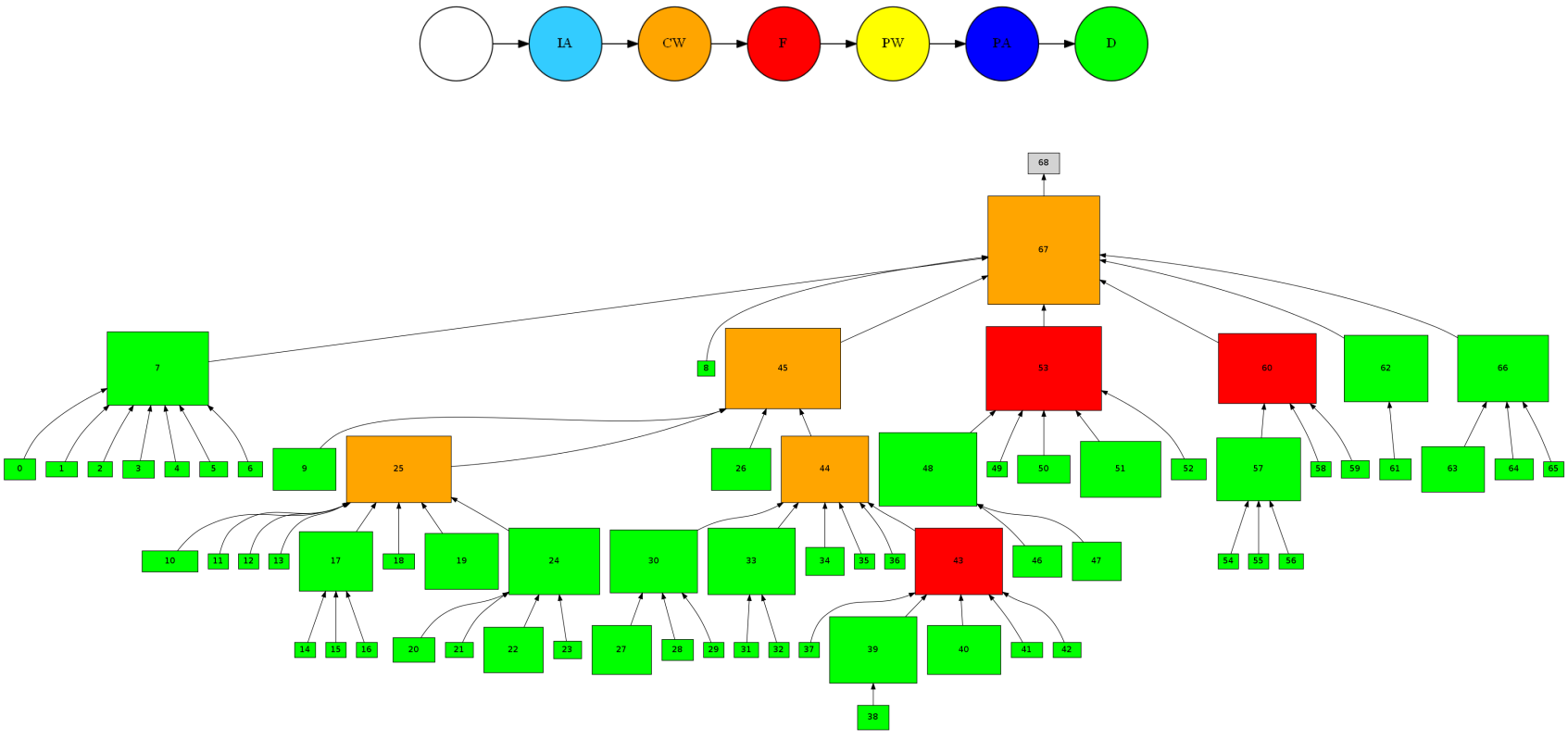

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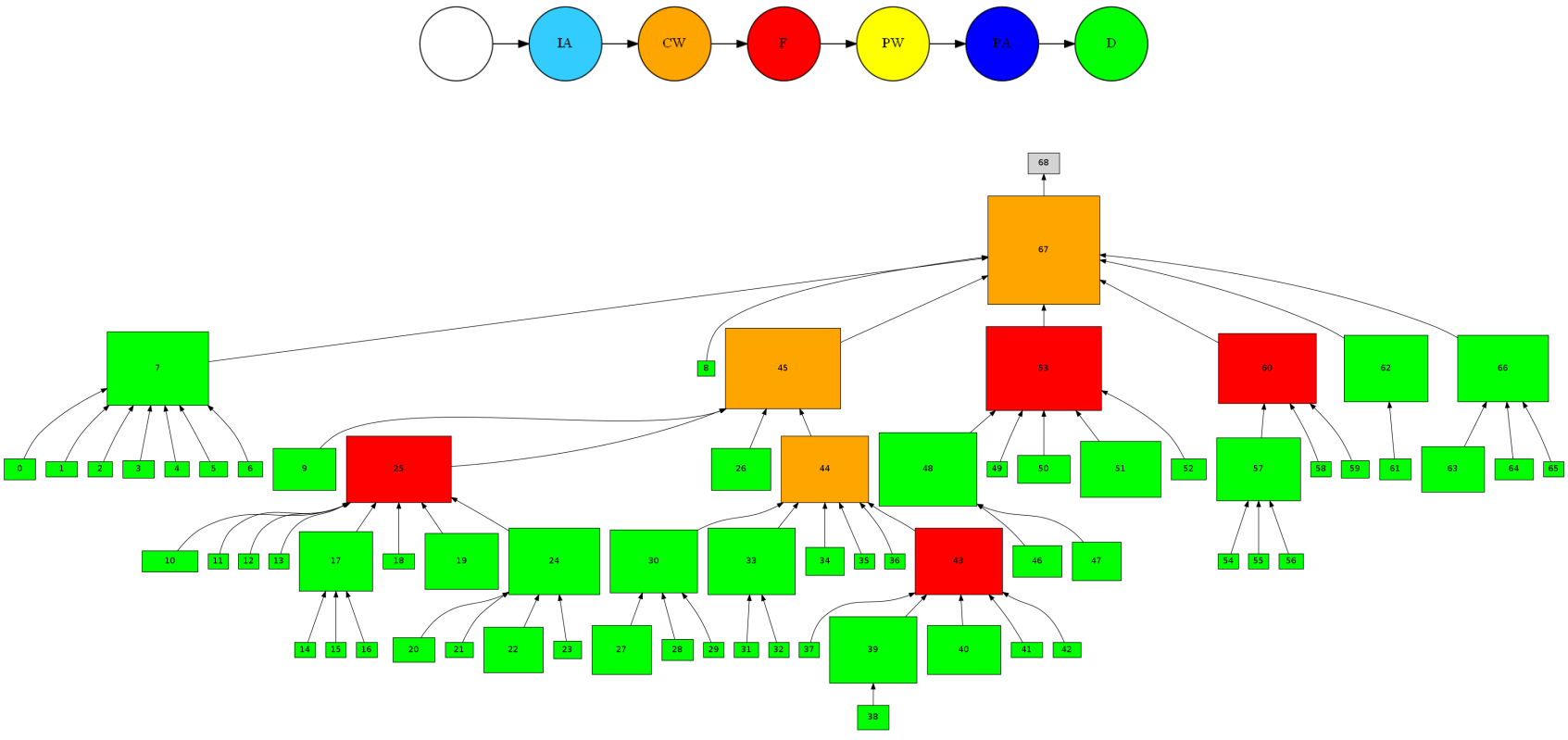

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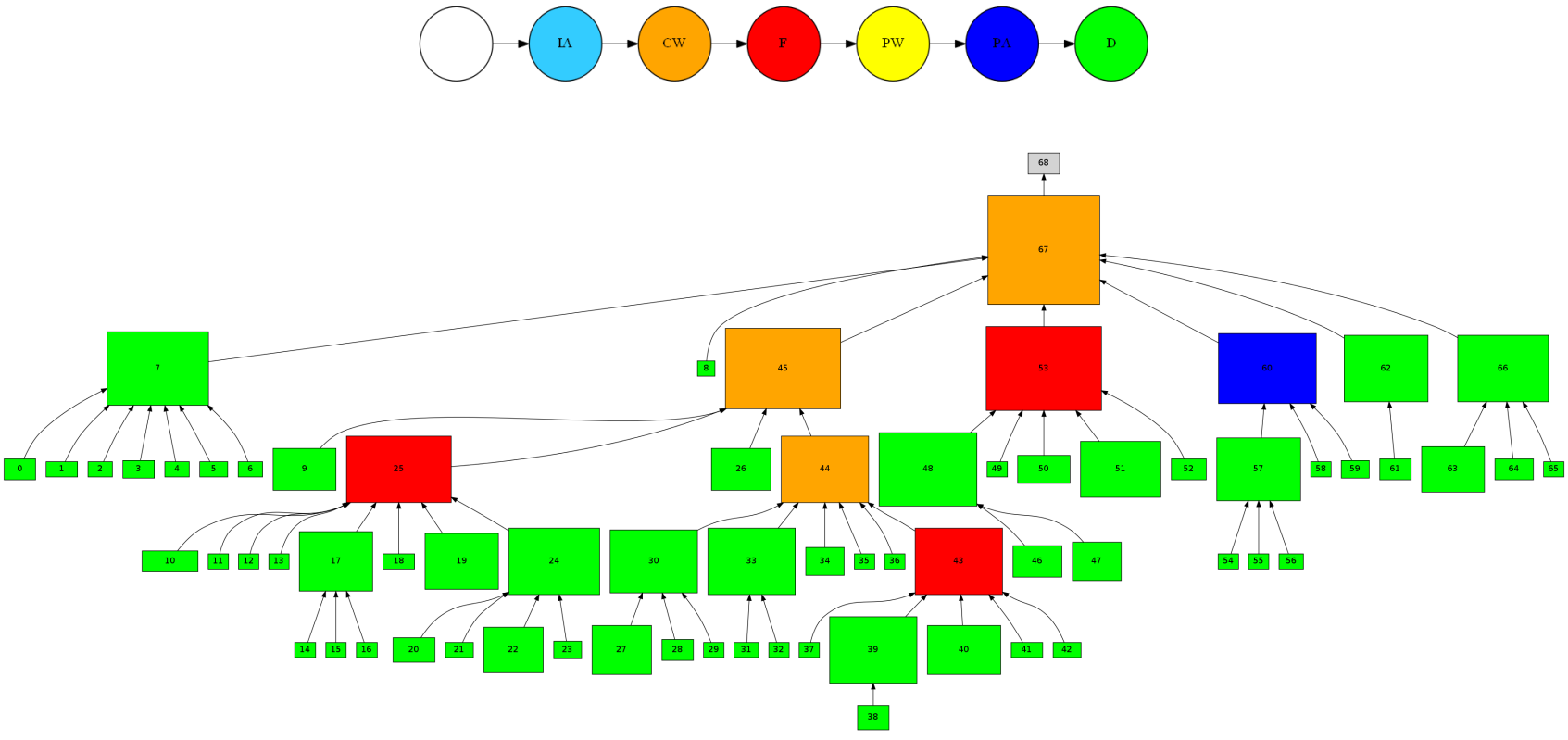

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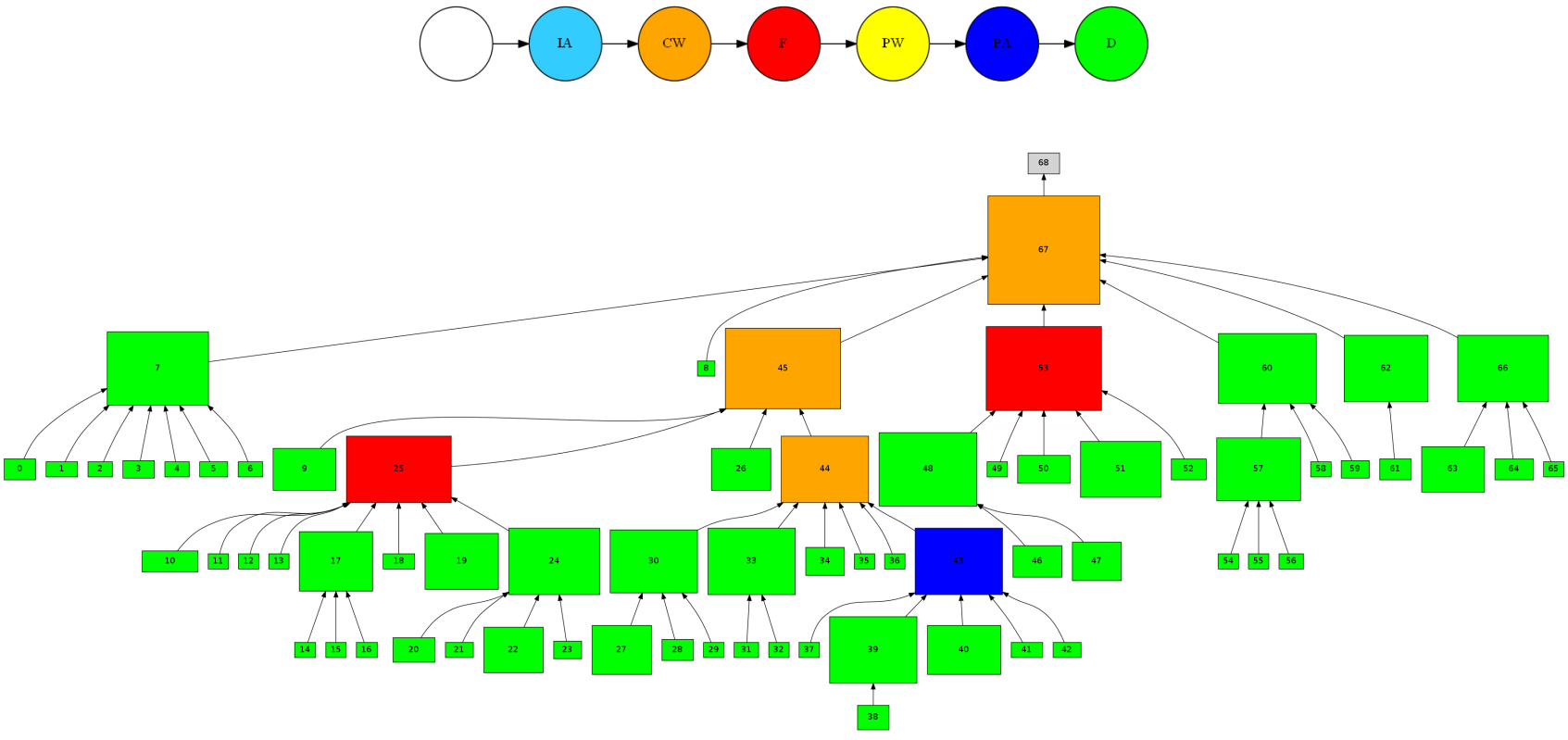

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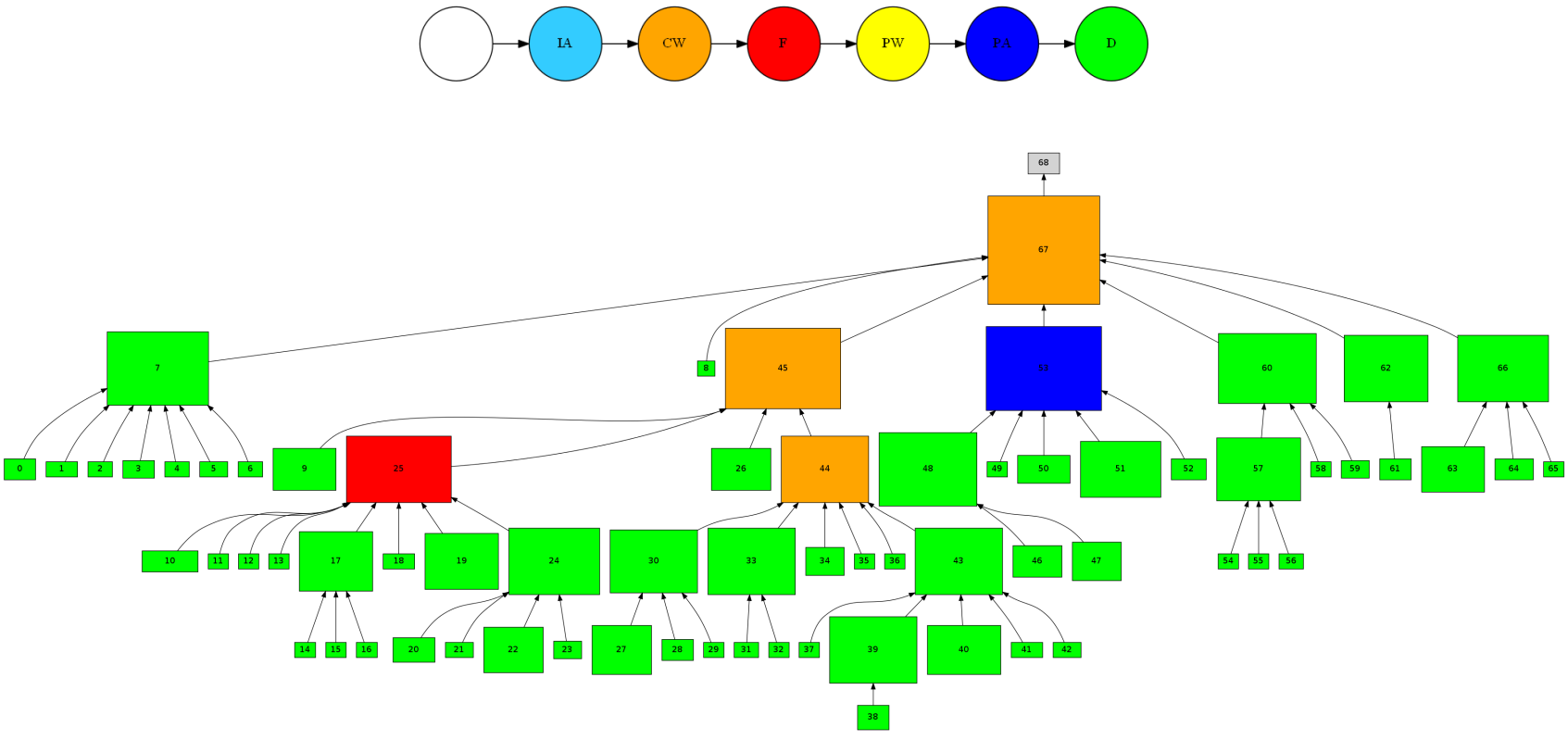

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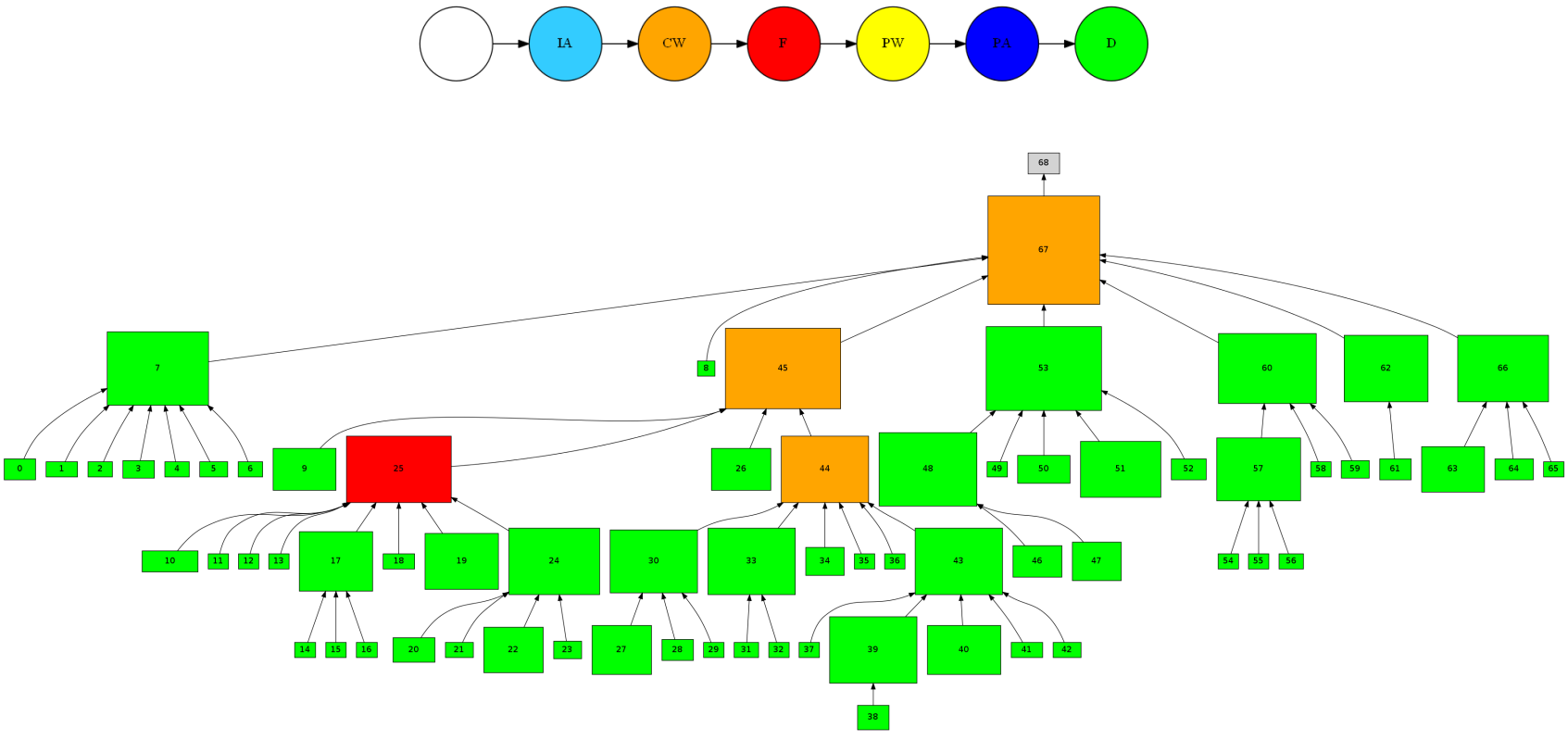

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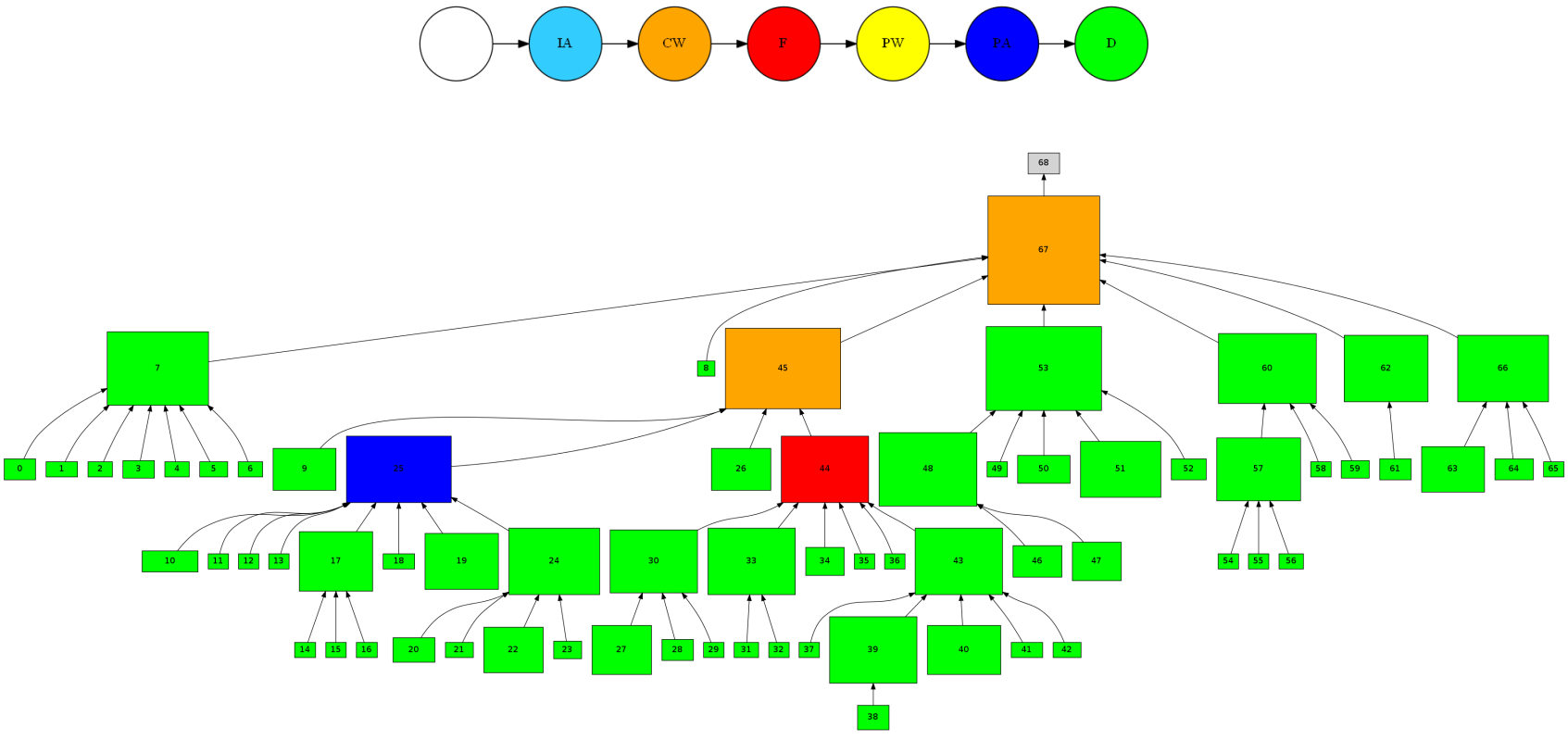

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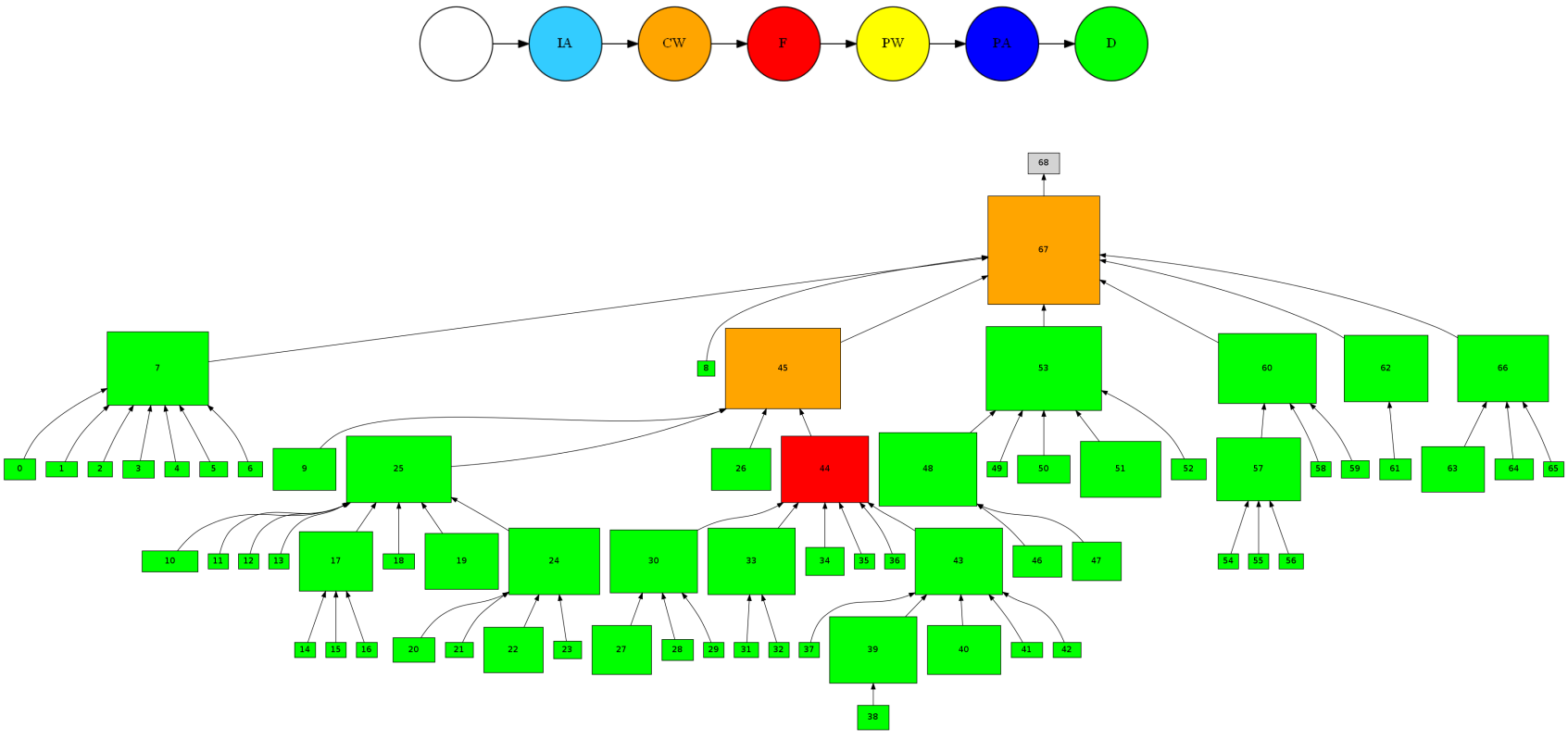

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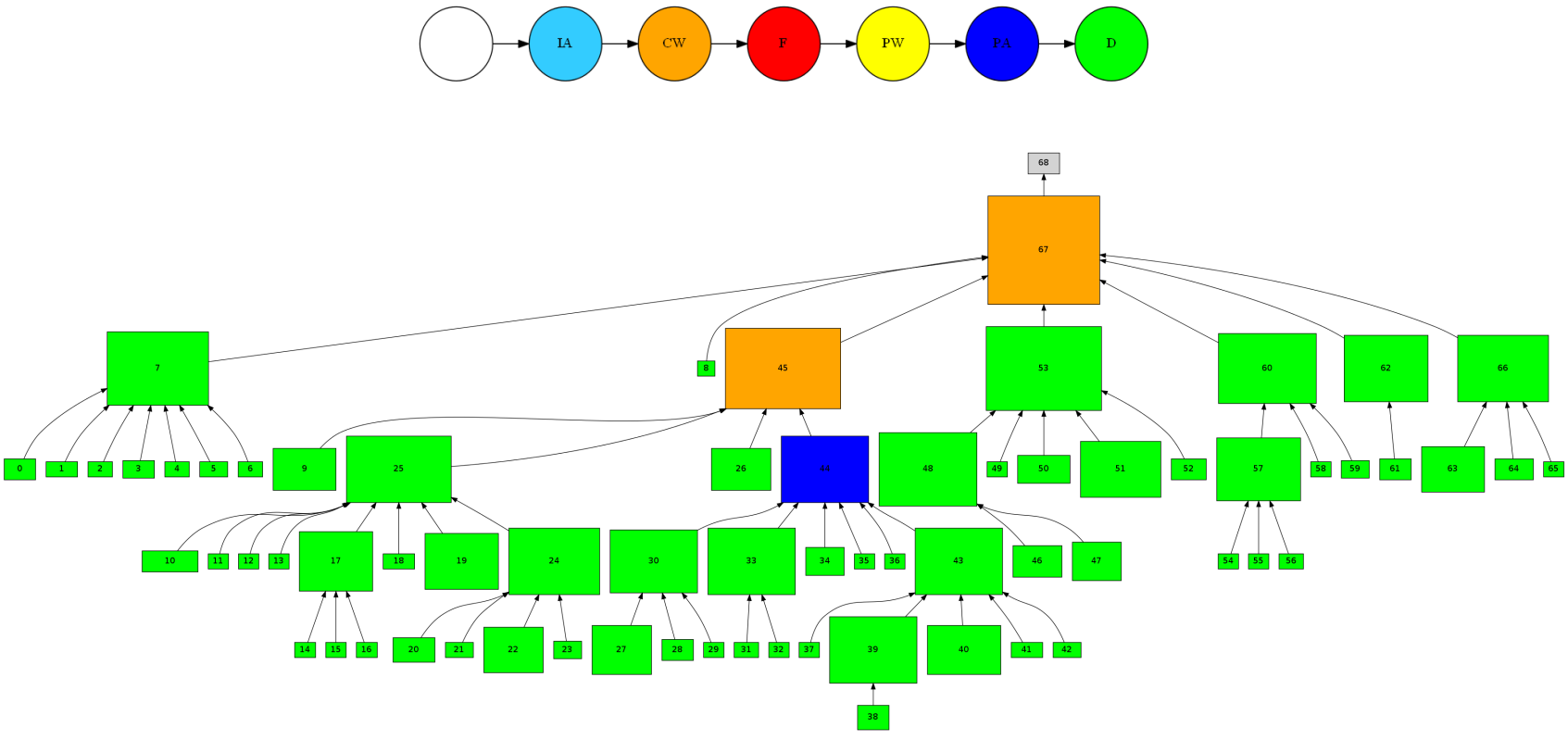

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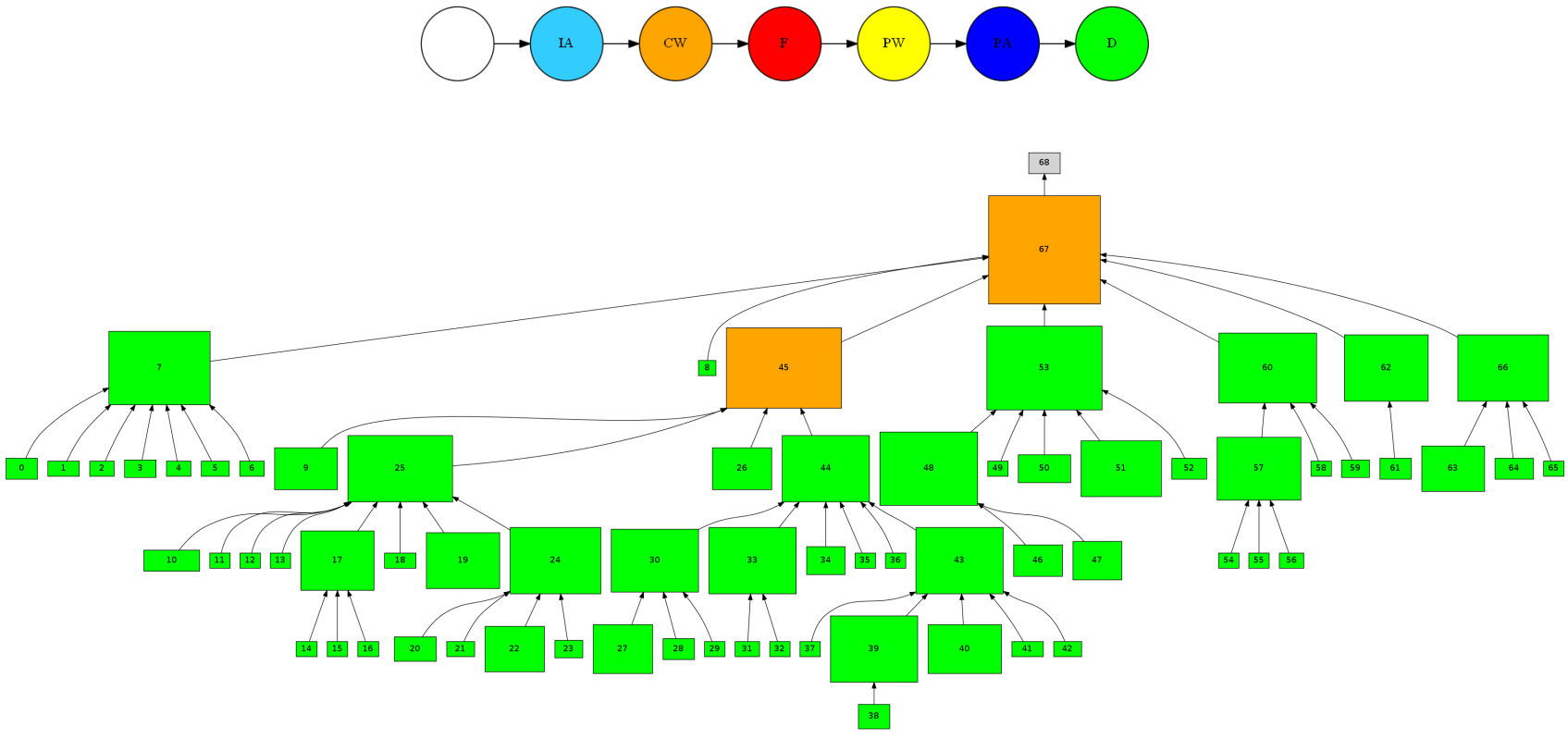

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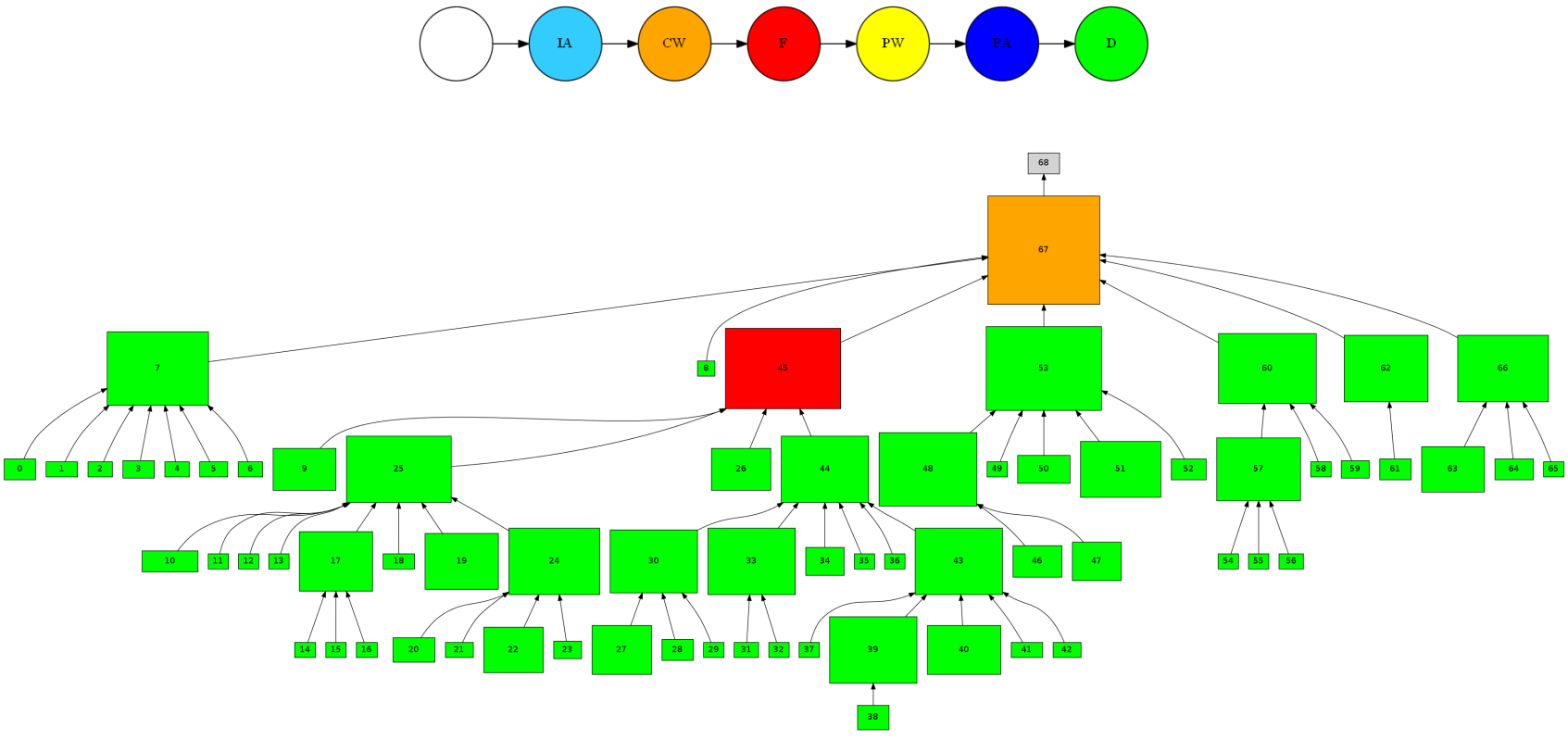

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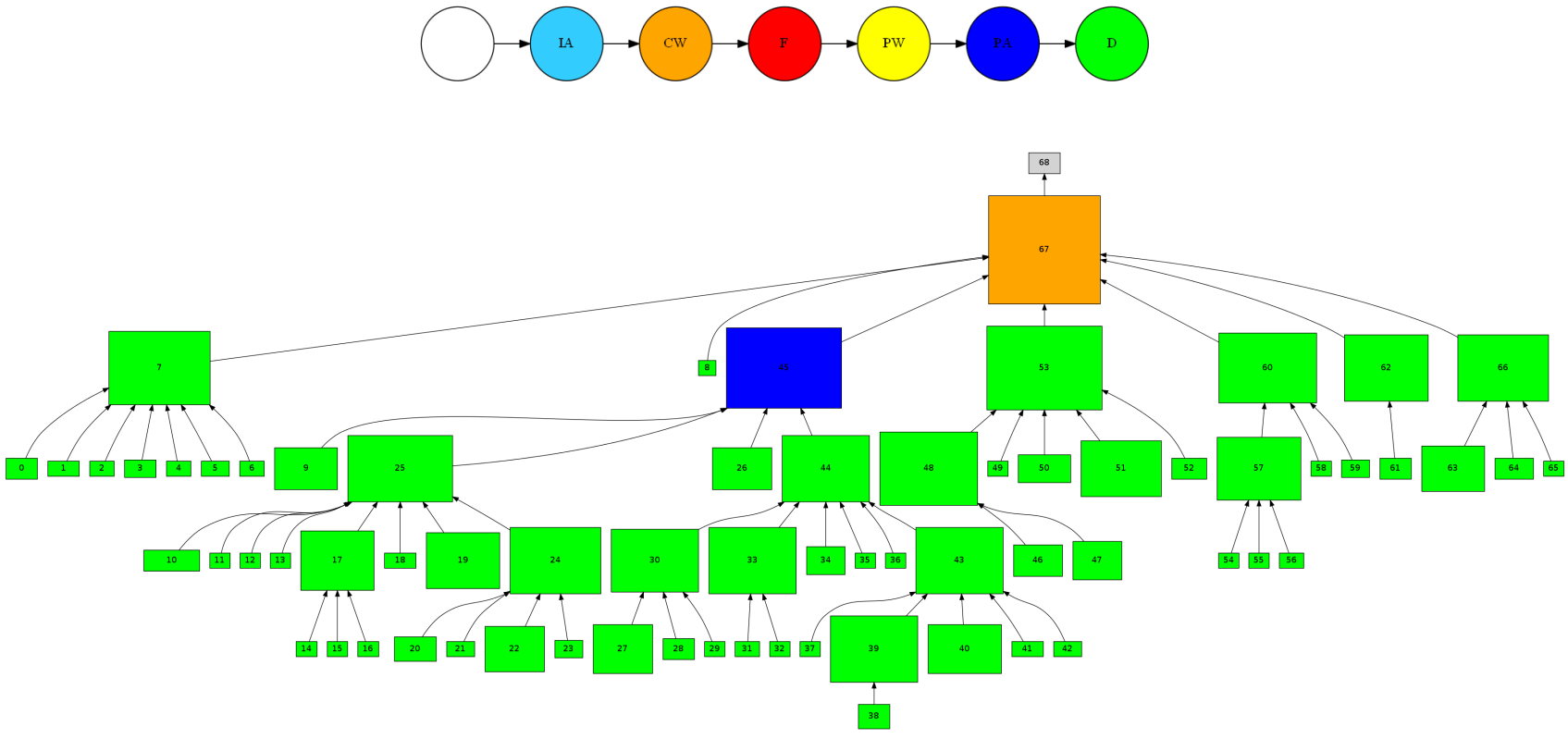

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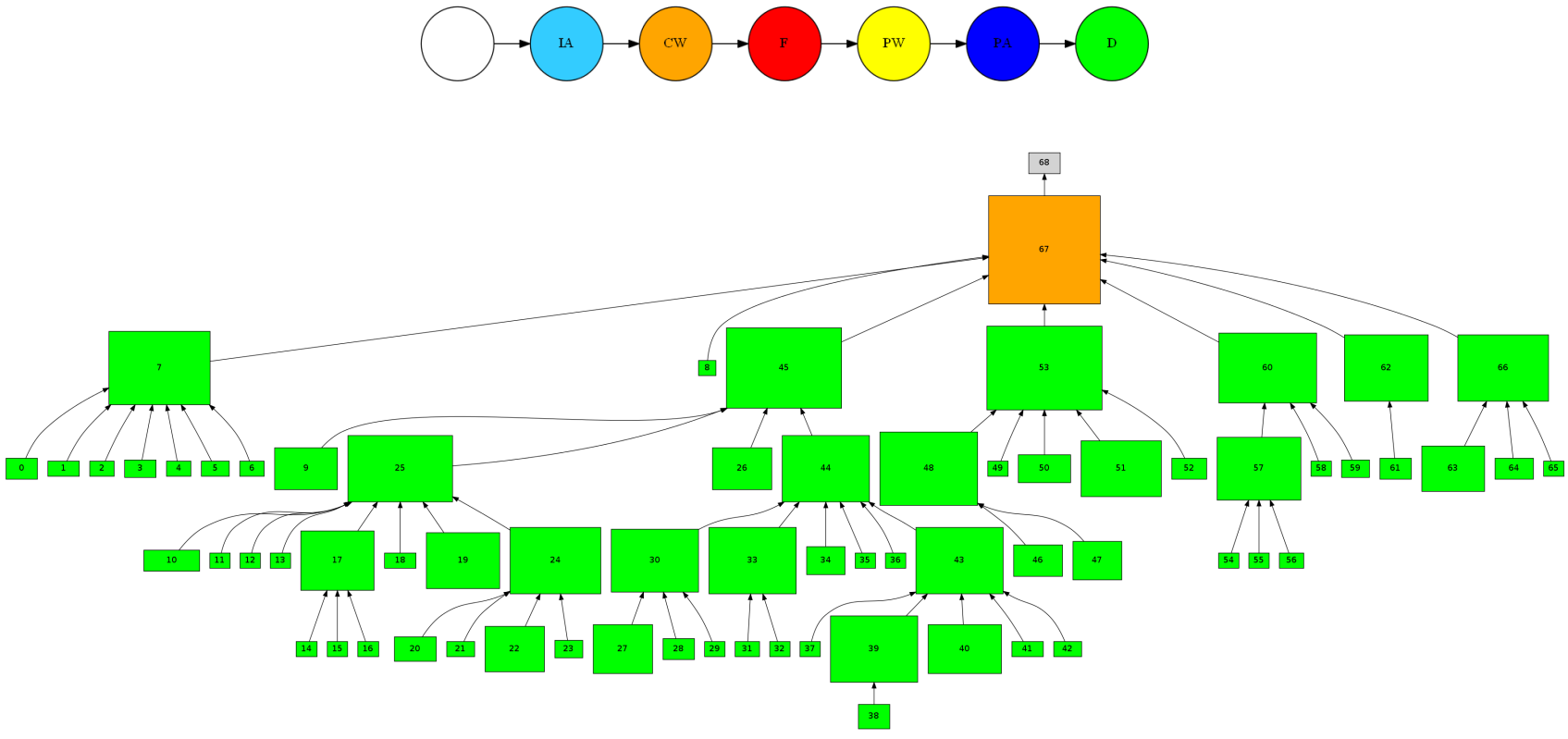

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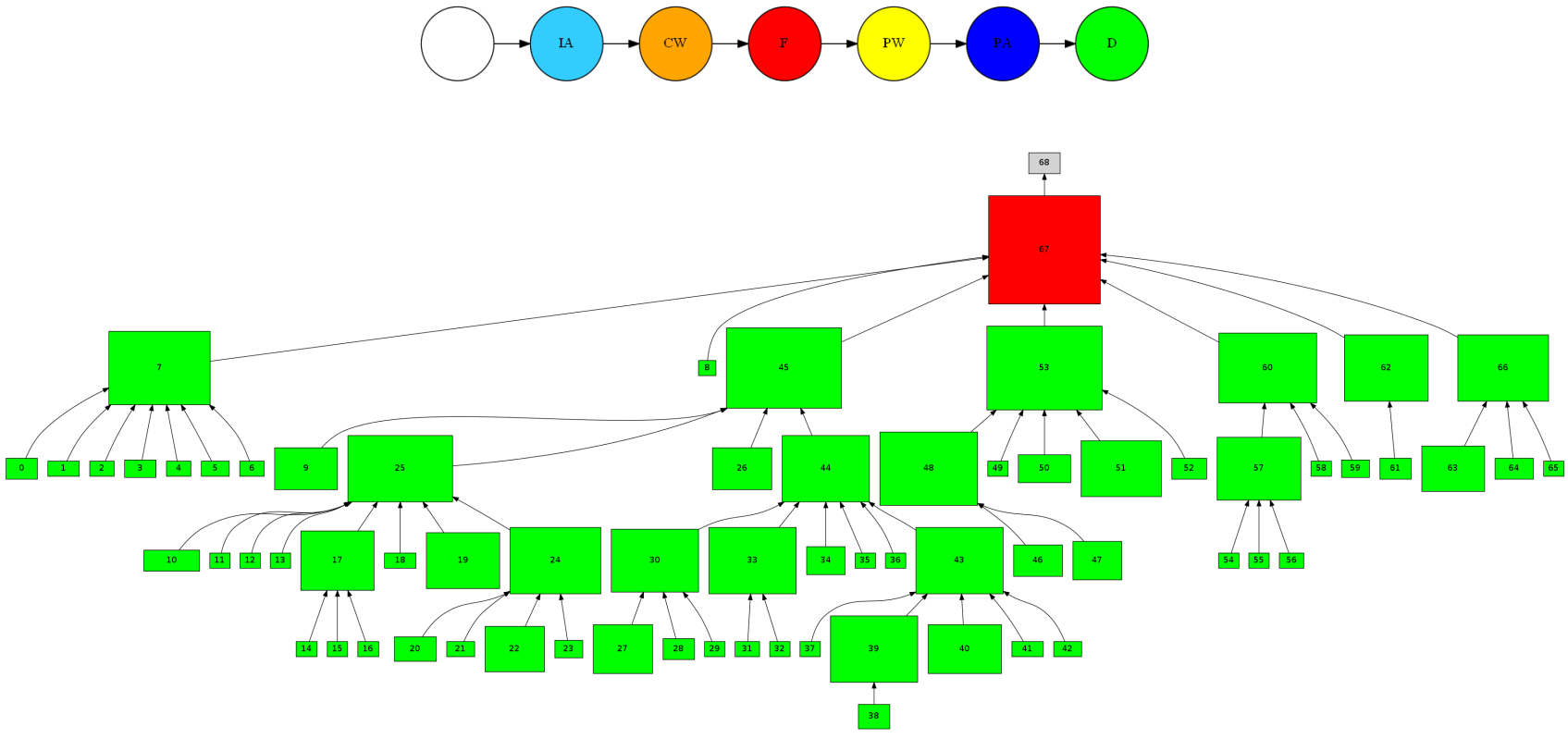

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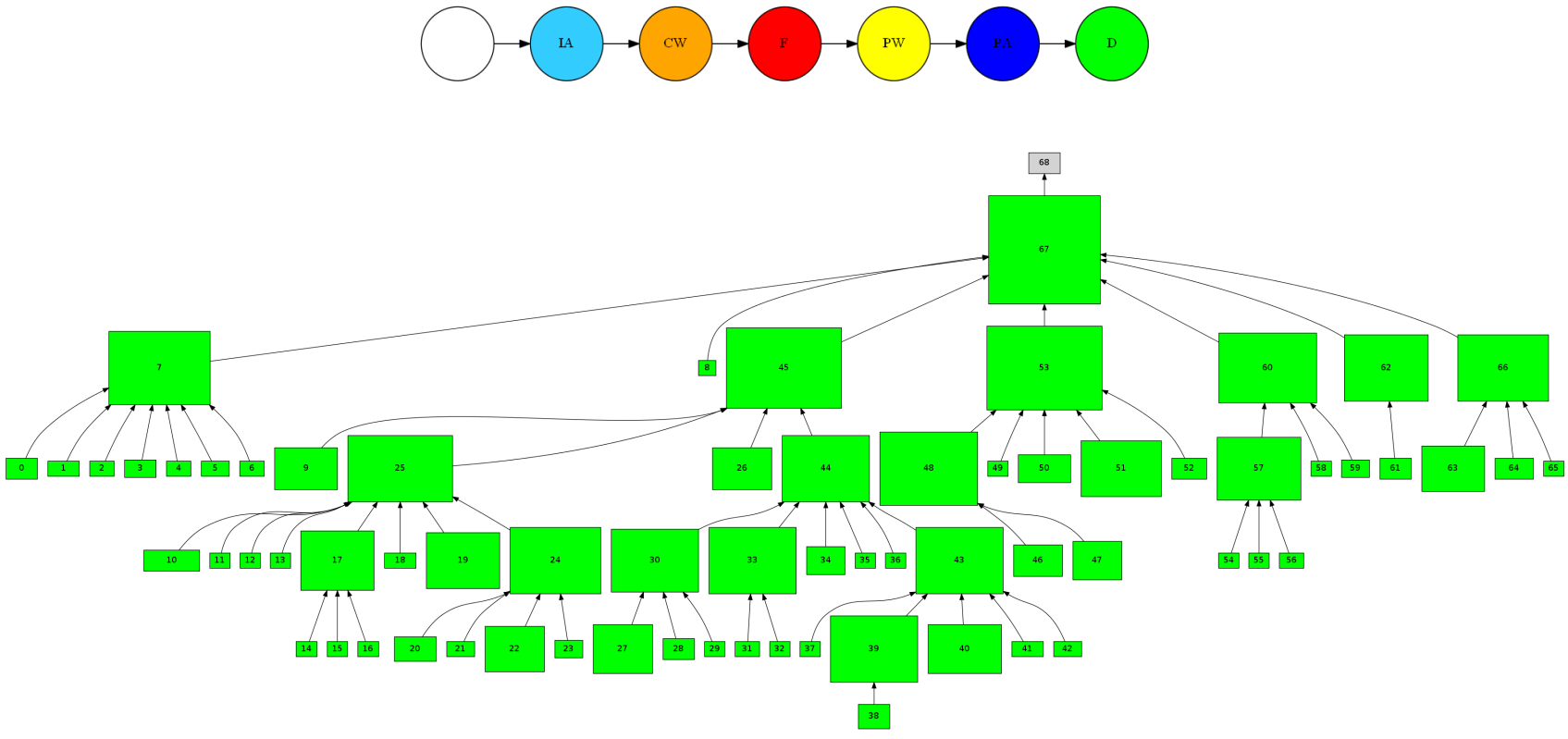
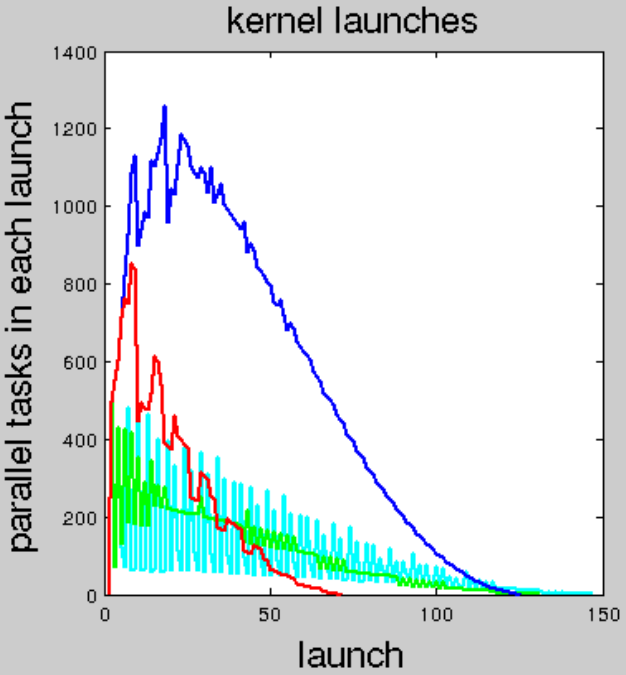
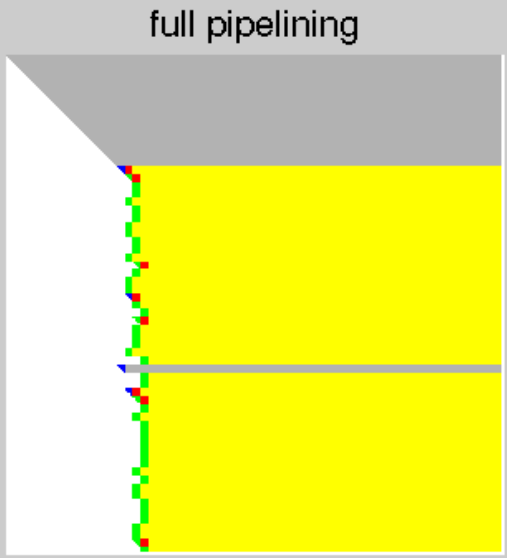
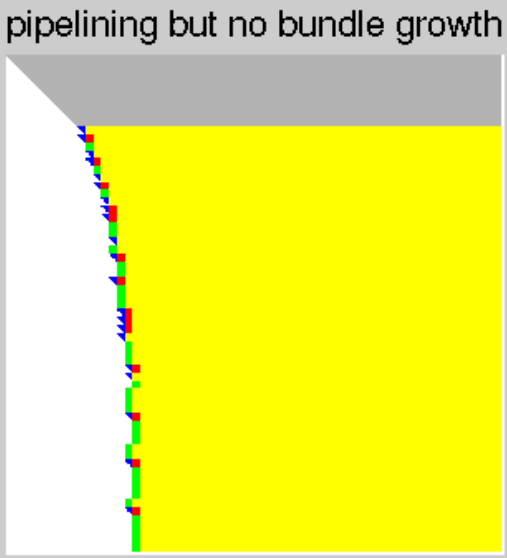


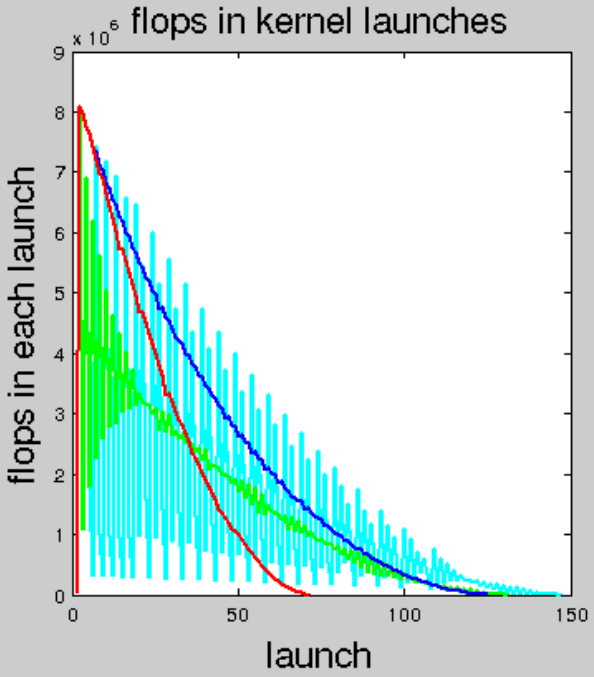
Pipelined Dense QR Factorization

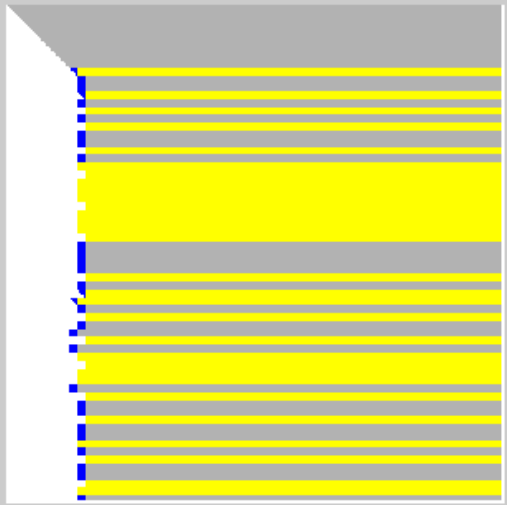
- **Results are coming soon..**
- **Our existing implementation is not efficient**
  - About half as fast as existing methods
  - Not exploiting enough parallelism!
  - We eliminate column blocks in order from left to right
  - GPU workload and throughput "thrashes," negating any gains from overlapping communication with computation
  - See BatchQR in CP7 for more details

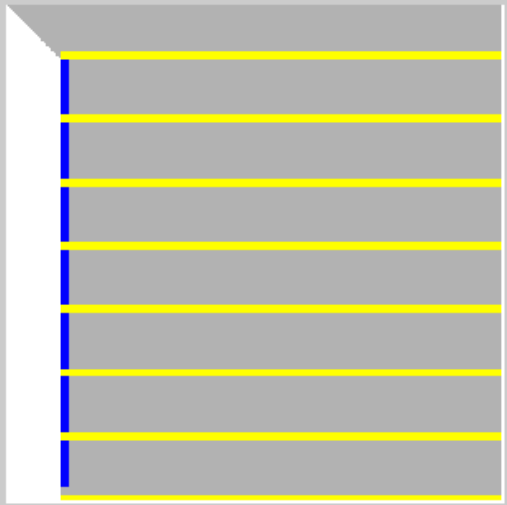

kernel launches
parallel tasks in each launch
launch
0
200
400
600
800
1000
1200
1400
0
50
100
150


full pipelining


pipelining but no bundle growth


flops in kernel launches
x 10^6
flops in each launch
launch
0
1
2
3
4
5
6
7
8
9
0
50
100
150


no pipelining


no pipelining, no lookahead

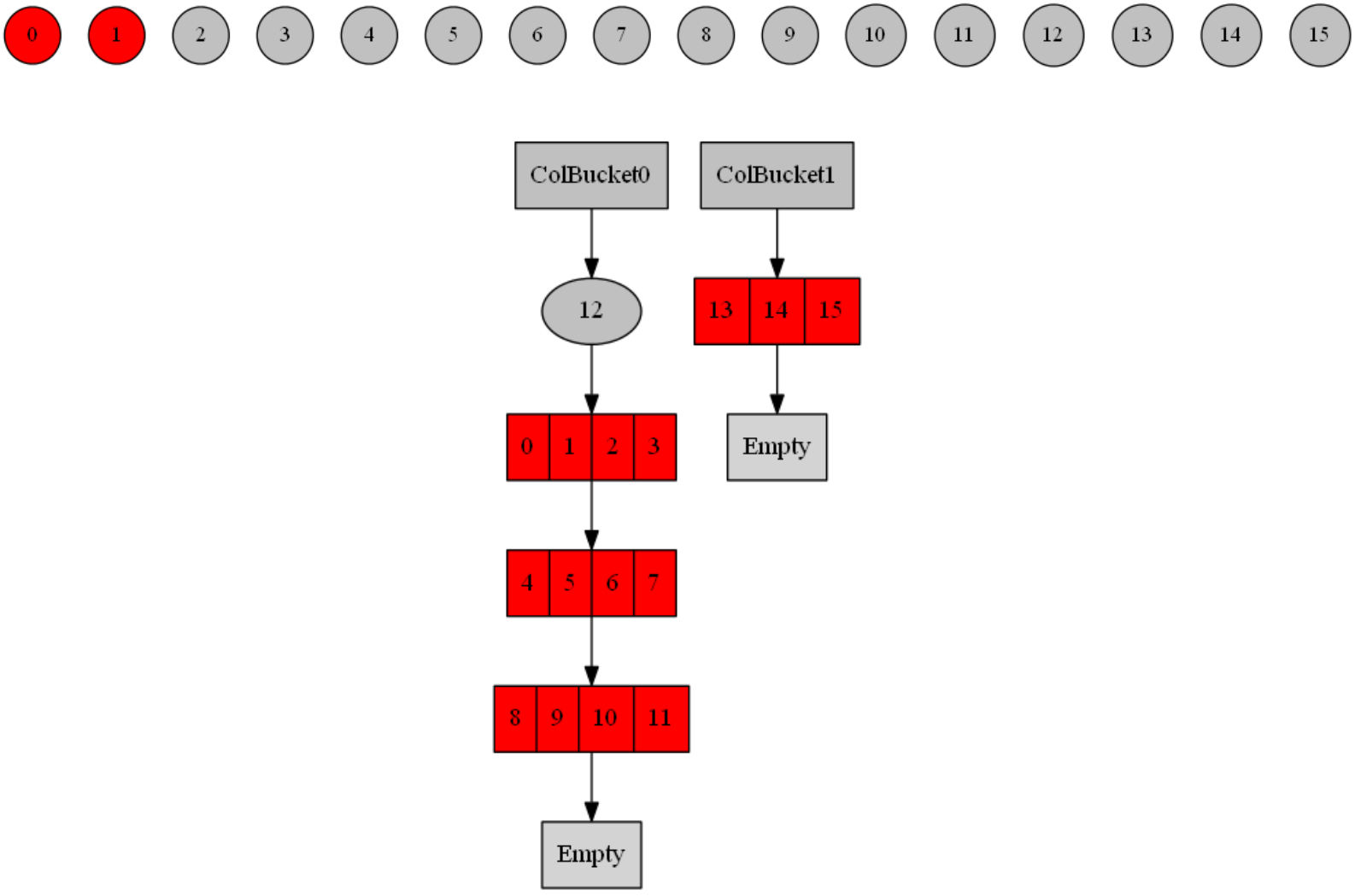
Pipelined Dense QR Factorization

- **Our current development thrust is promising**
  - Tile each front
  - Track each block row's leftmost tile
  - Initialize the tracking using the Staircase
  - Bundle tiles into packets of parallel tasks
  - Launch each GPU kernel with the most work we can do before we need to synchronize
  - Pipeline the block Householder applies with an annihilate

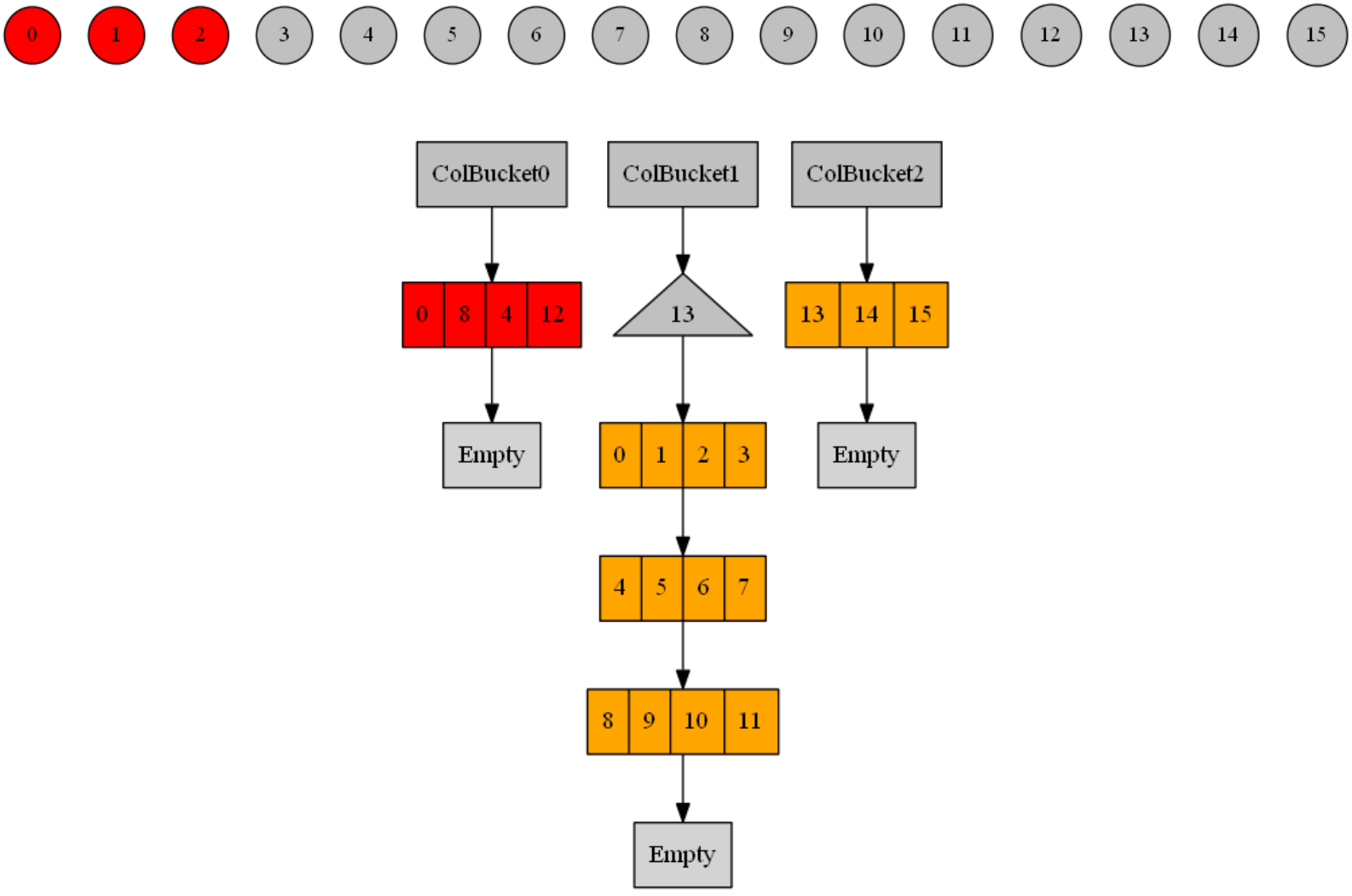
Pipelined Dense QR Factorization

- **What does this look like?**

Pipelined Factorization



Pipelined Factorization



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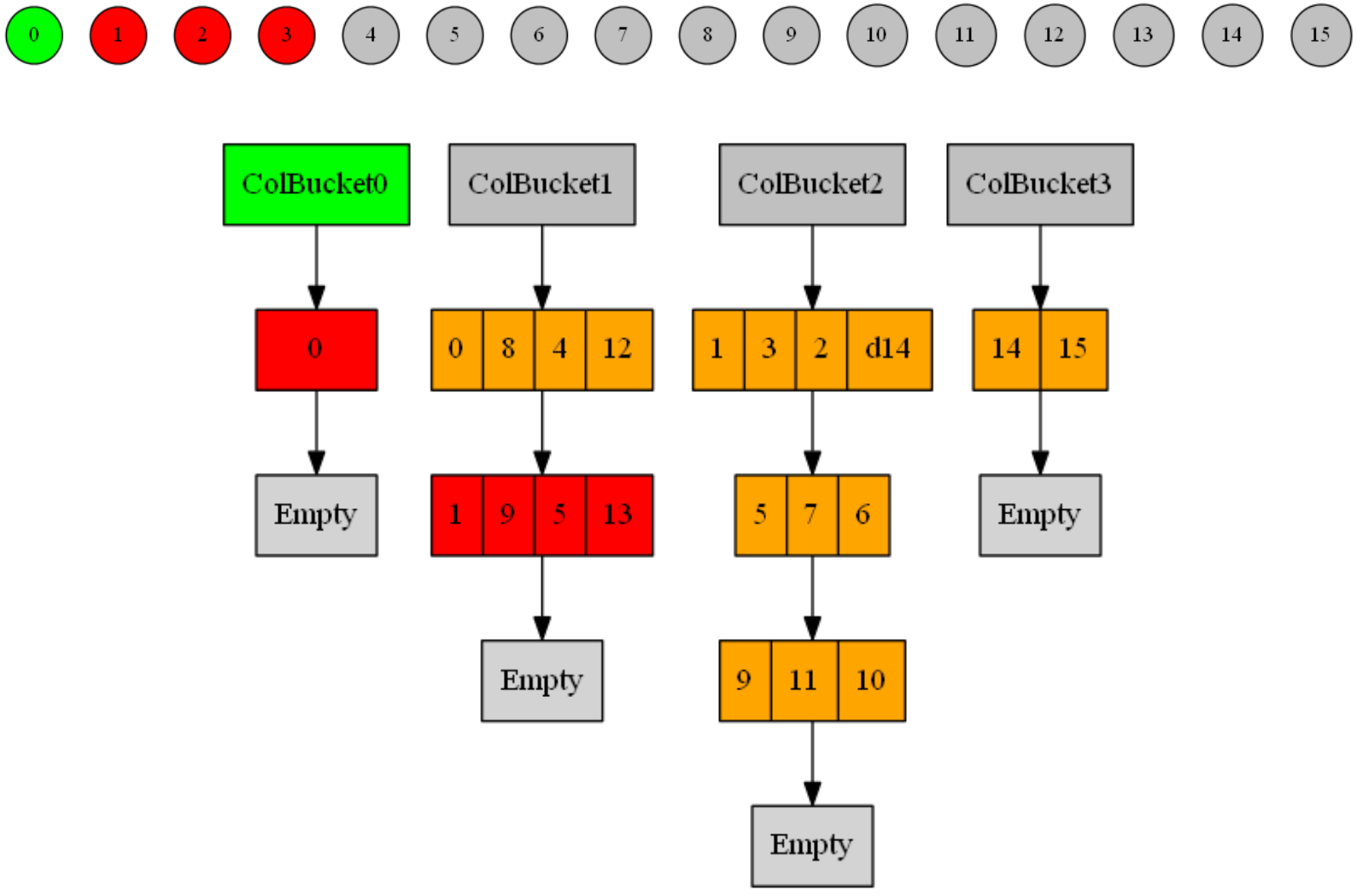

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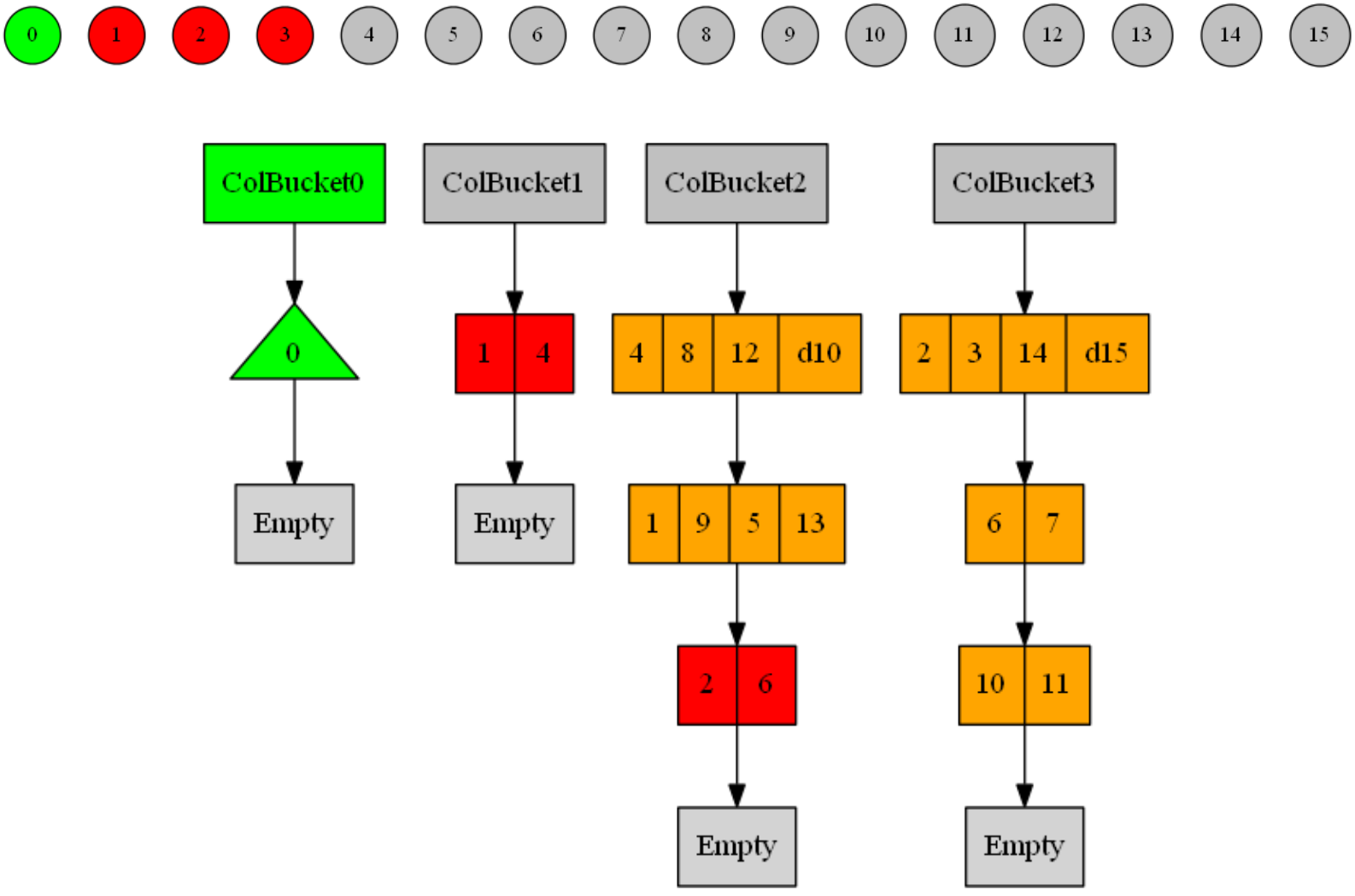

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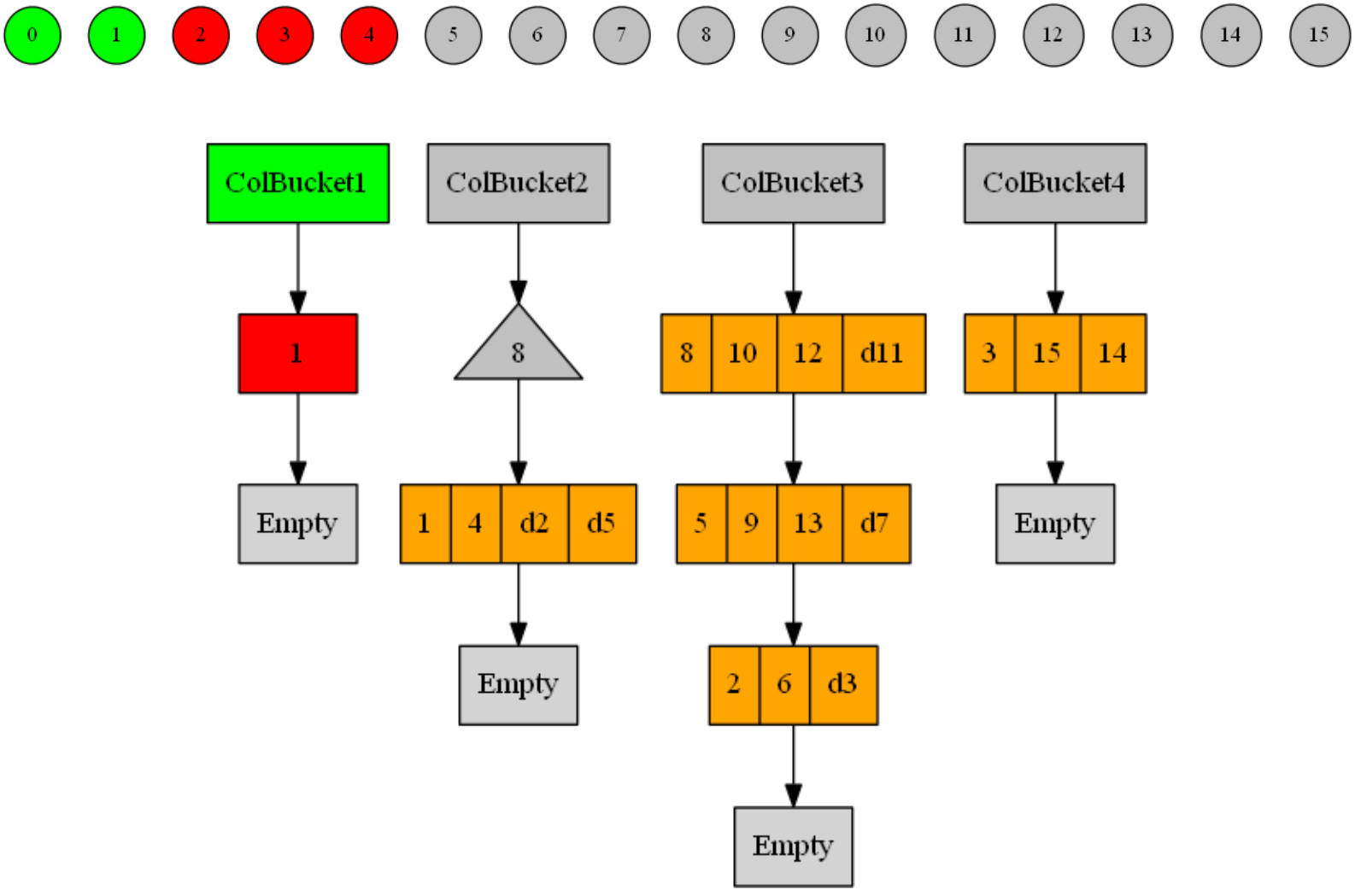

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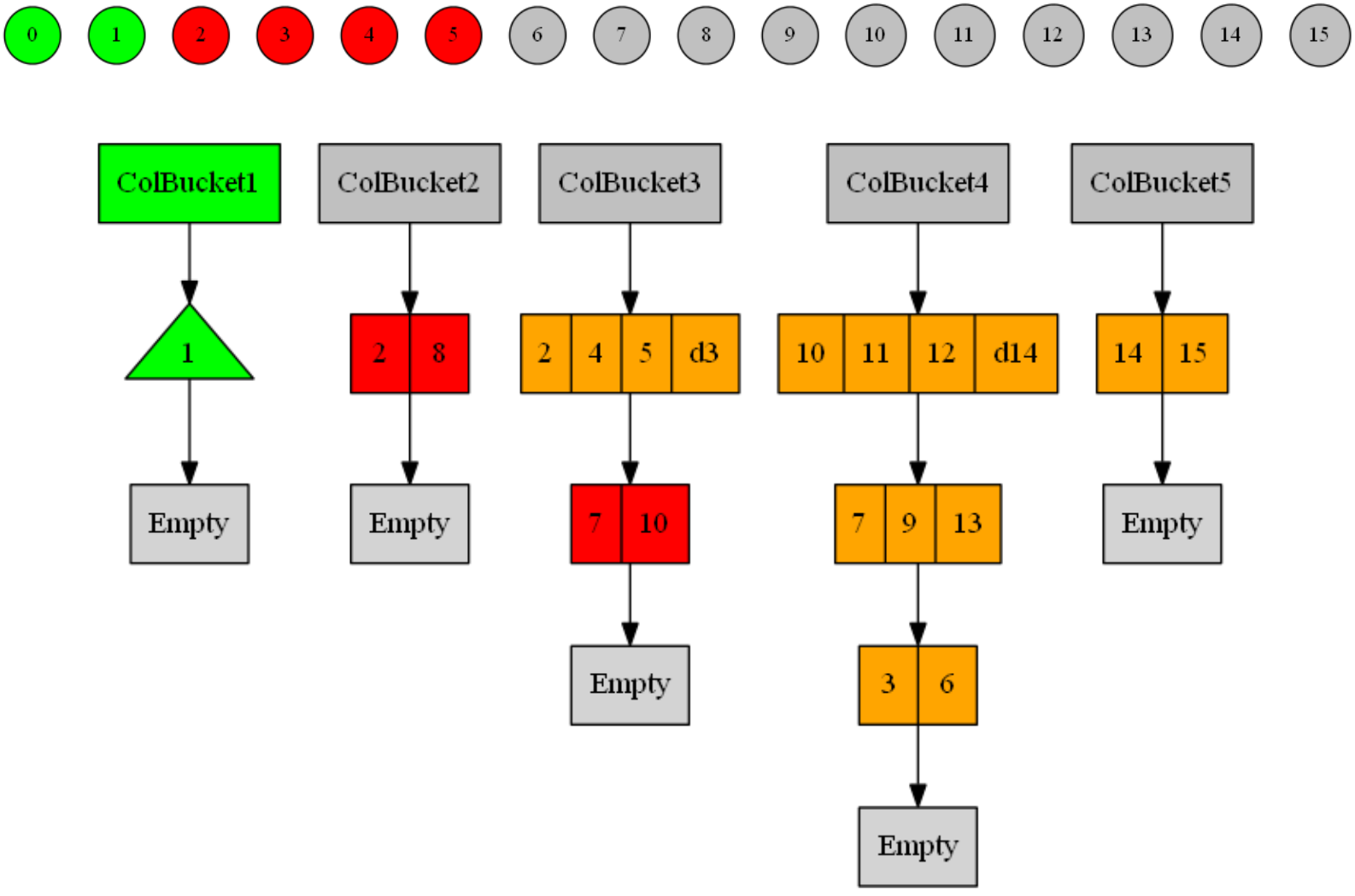

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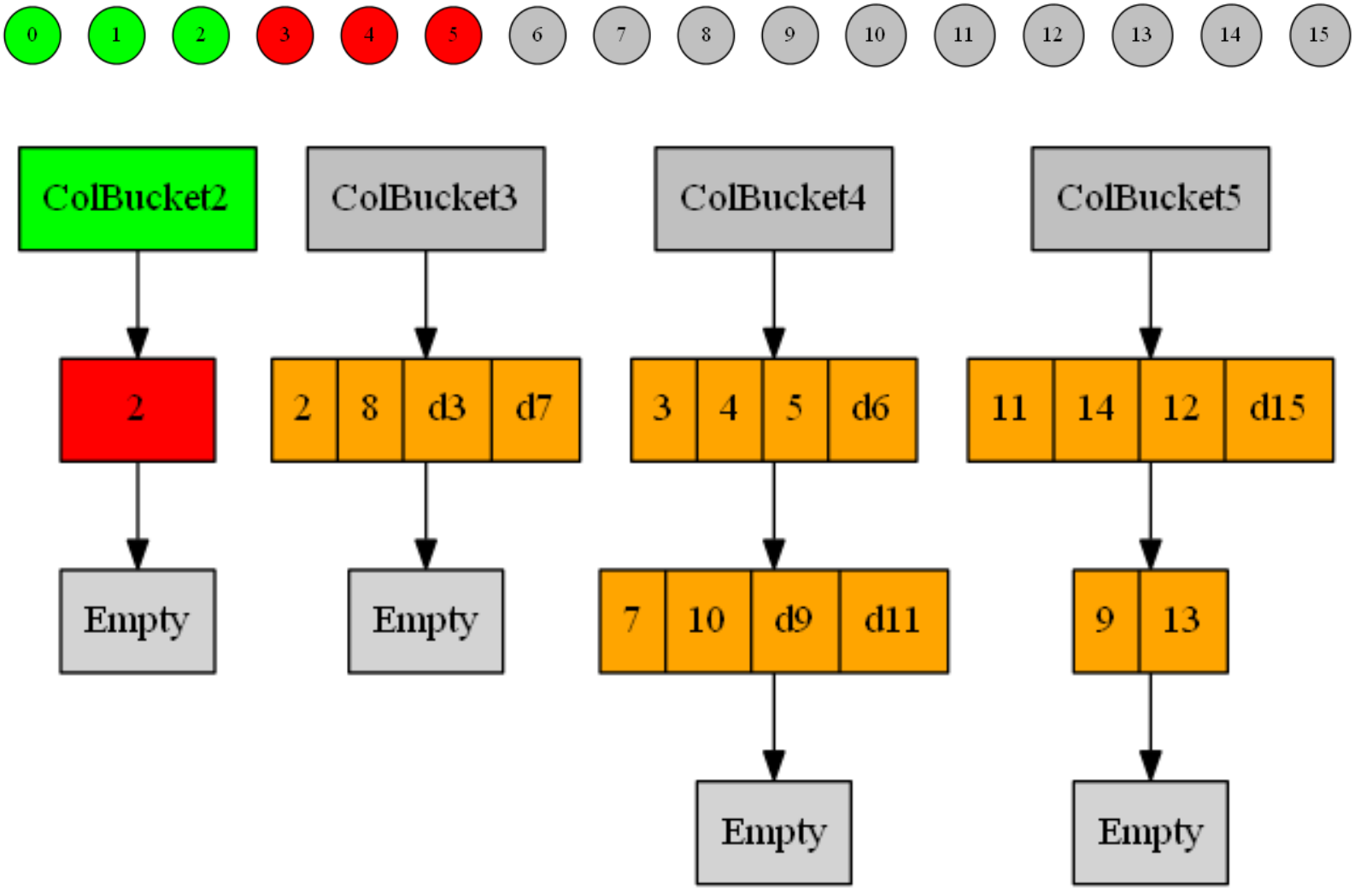

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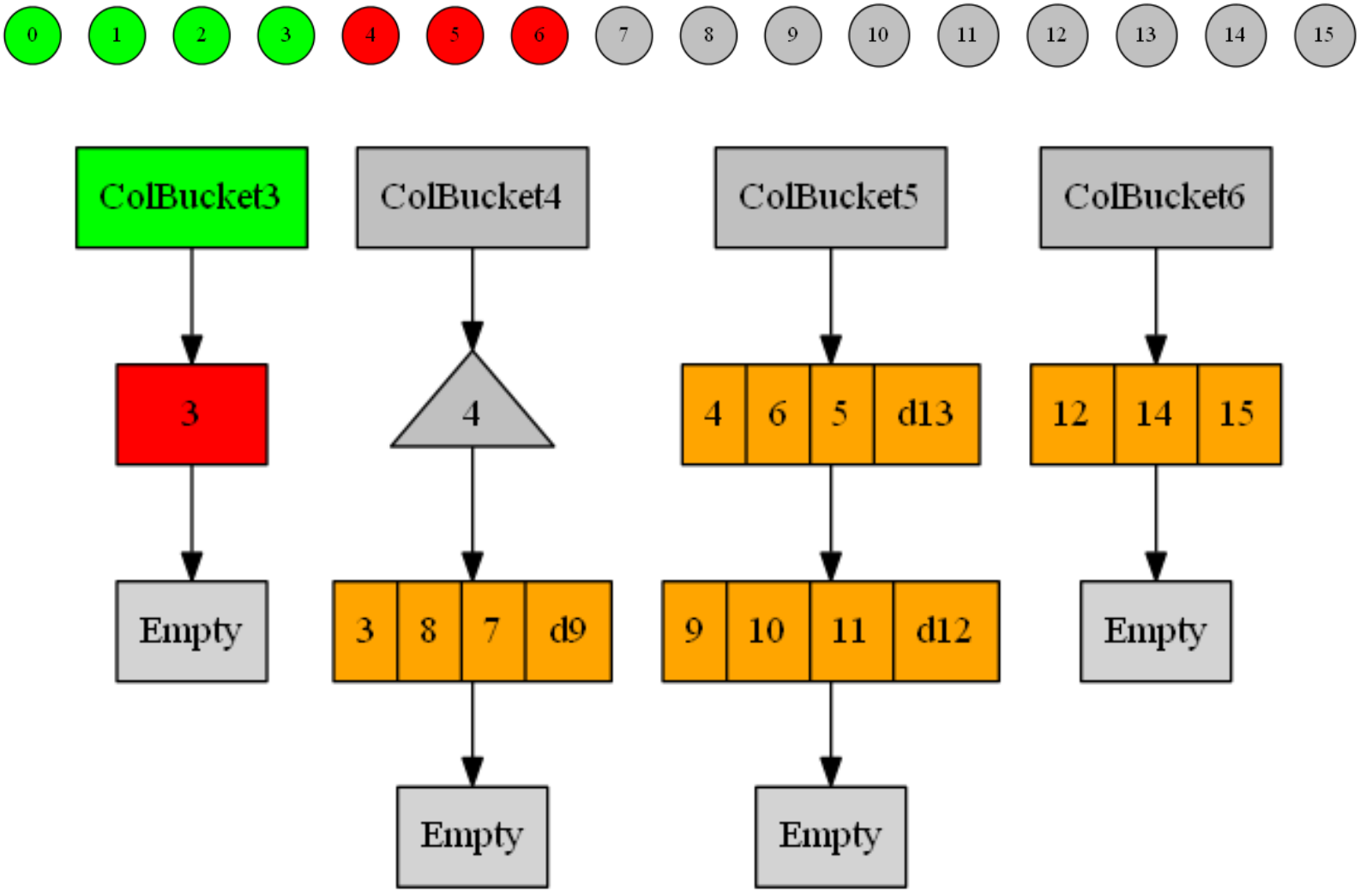

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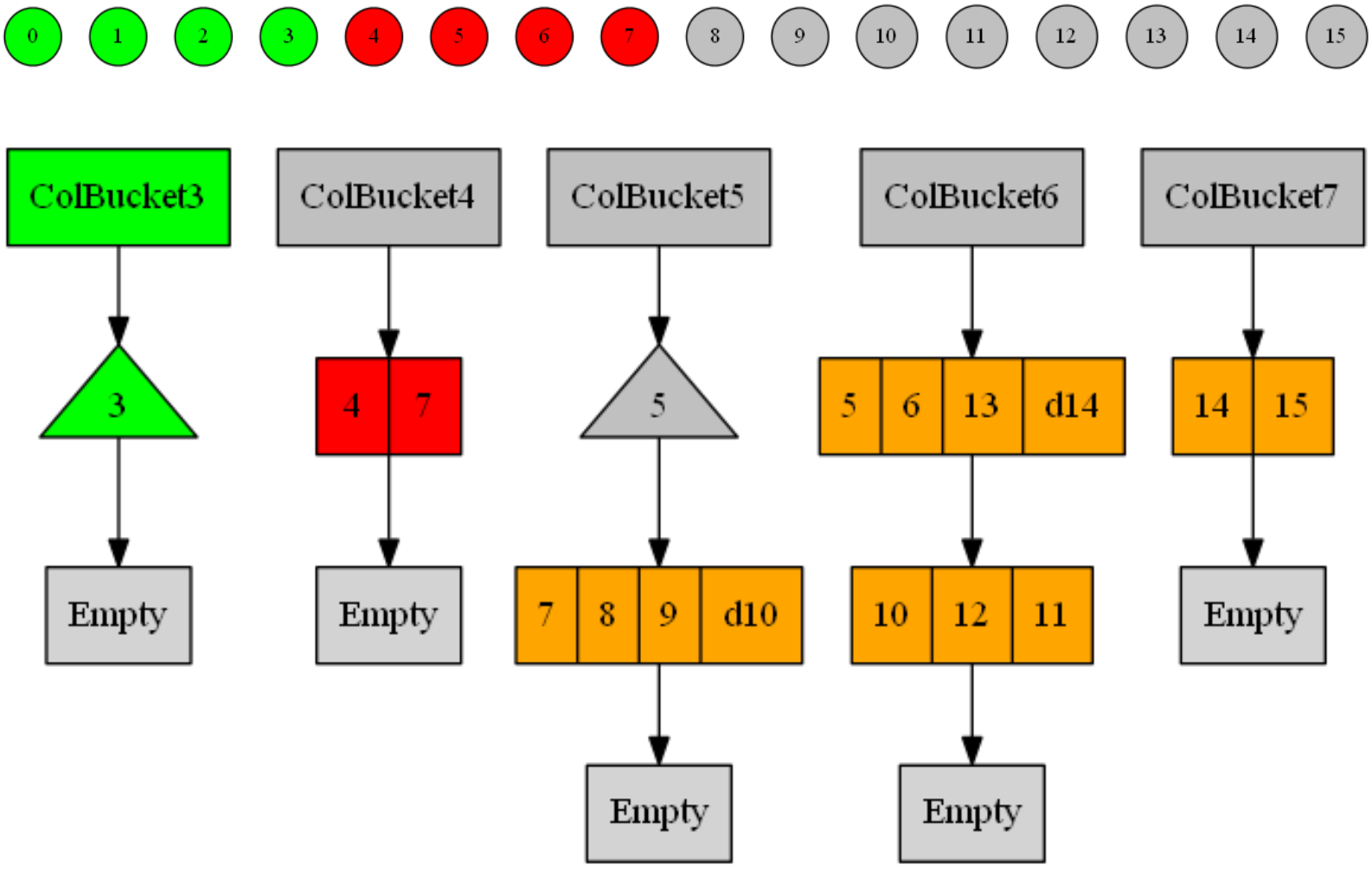

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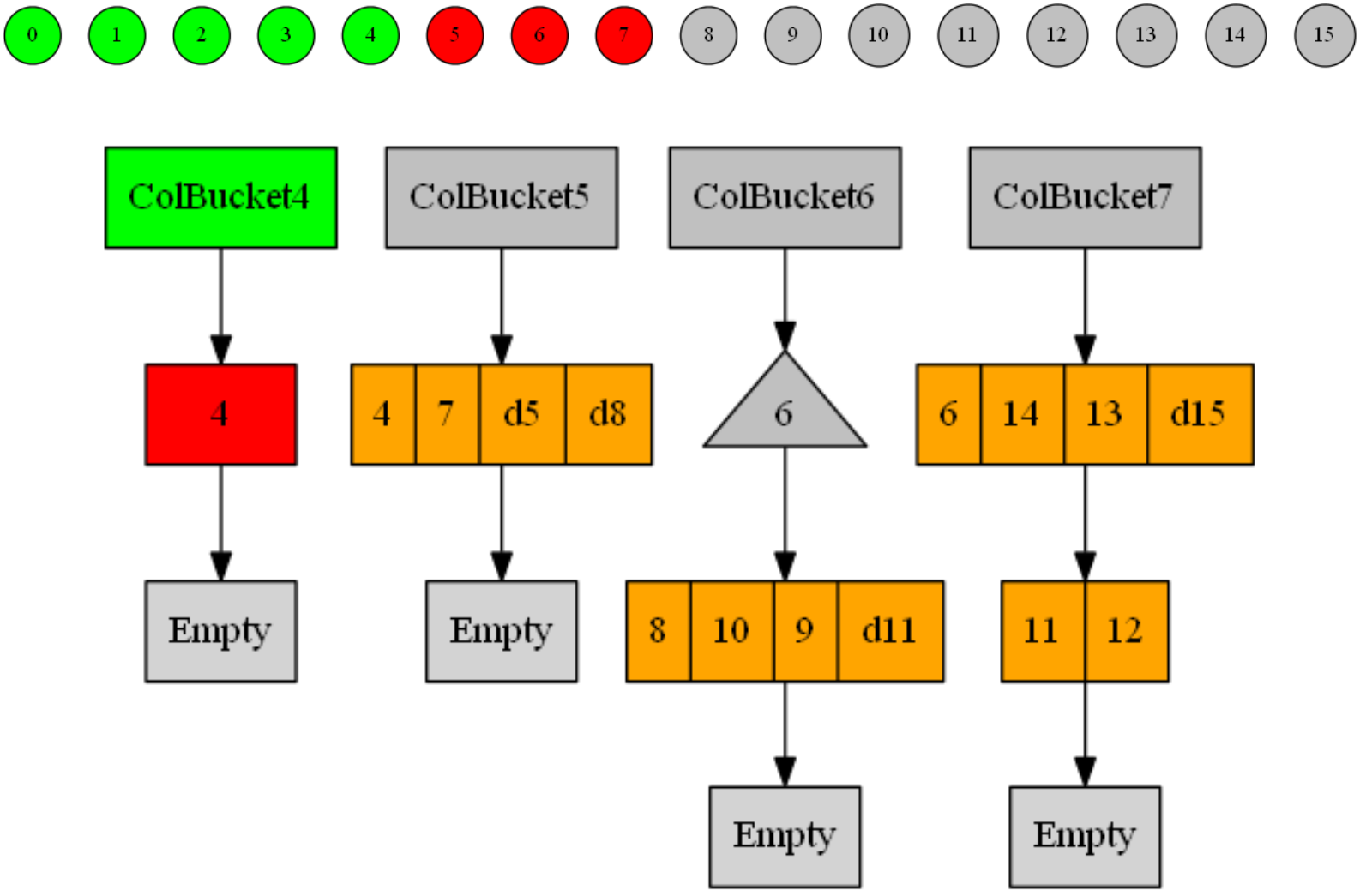

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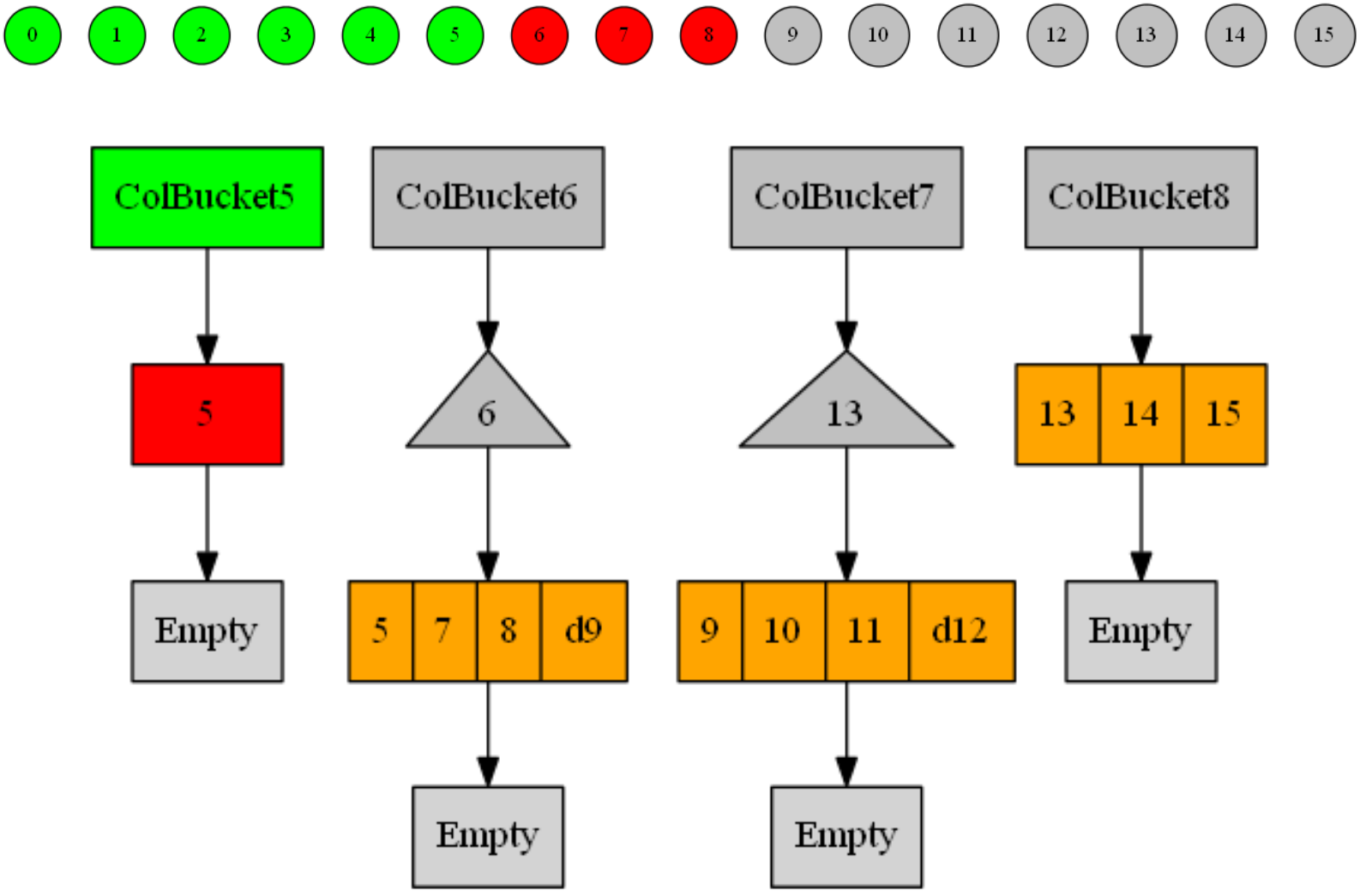

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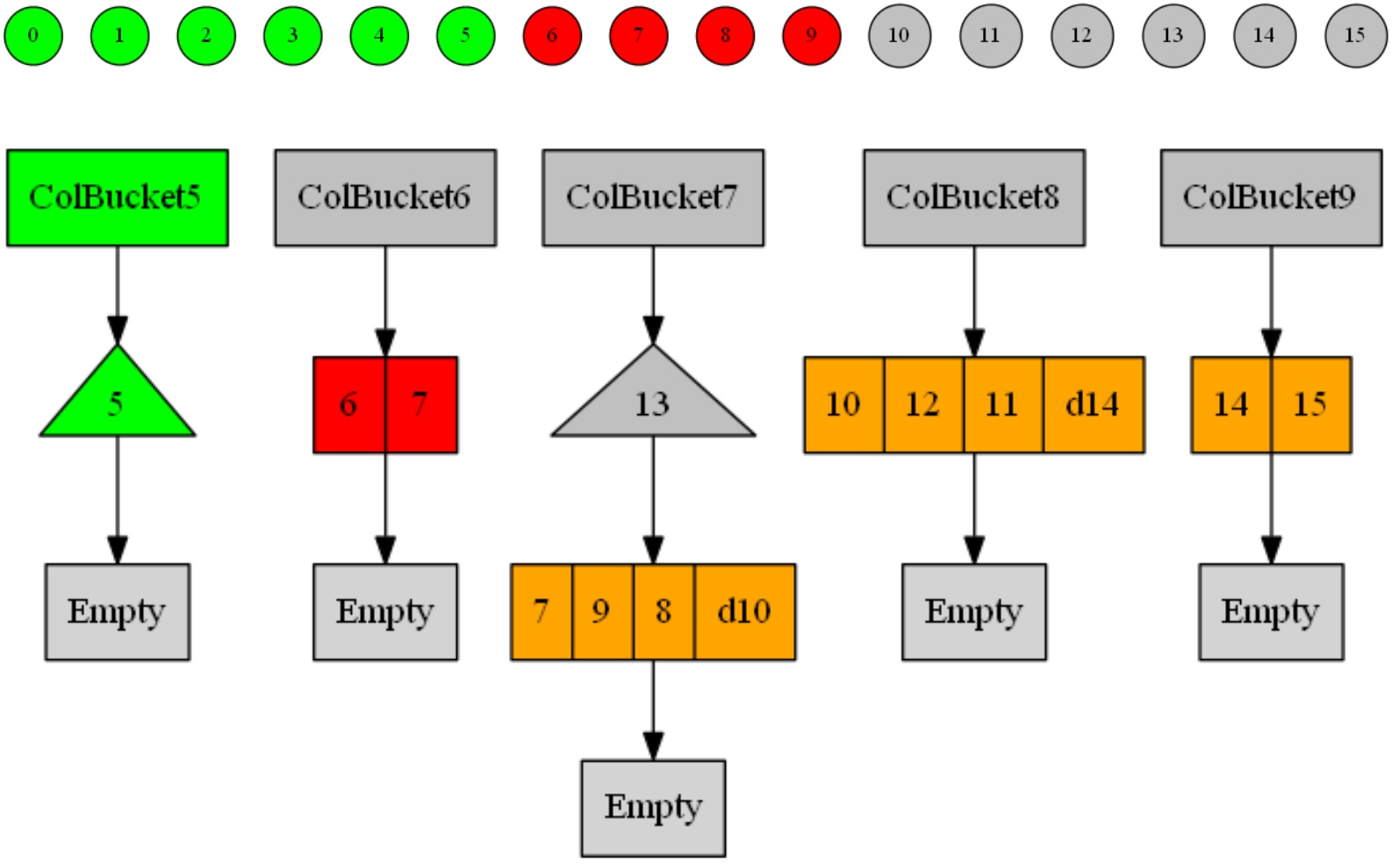

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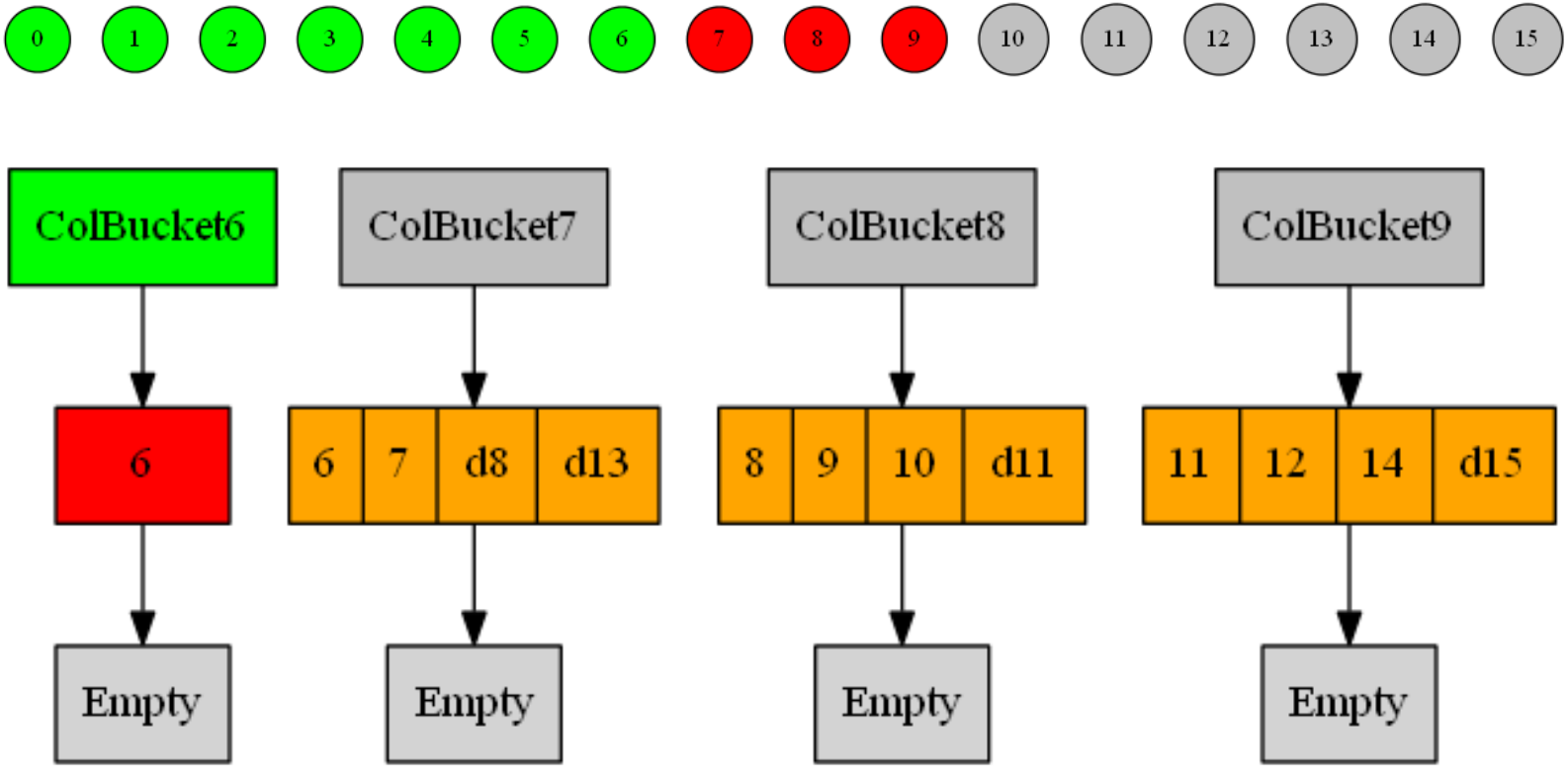

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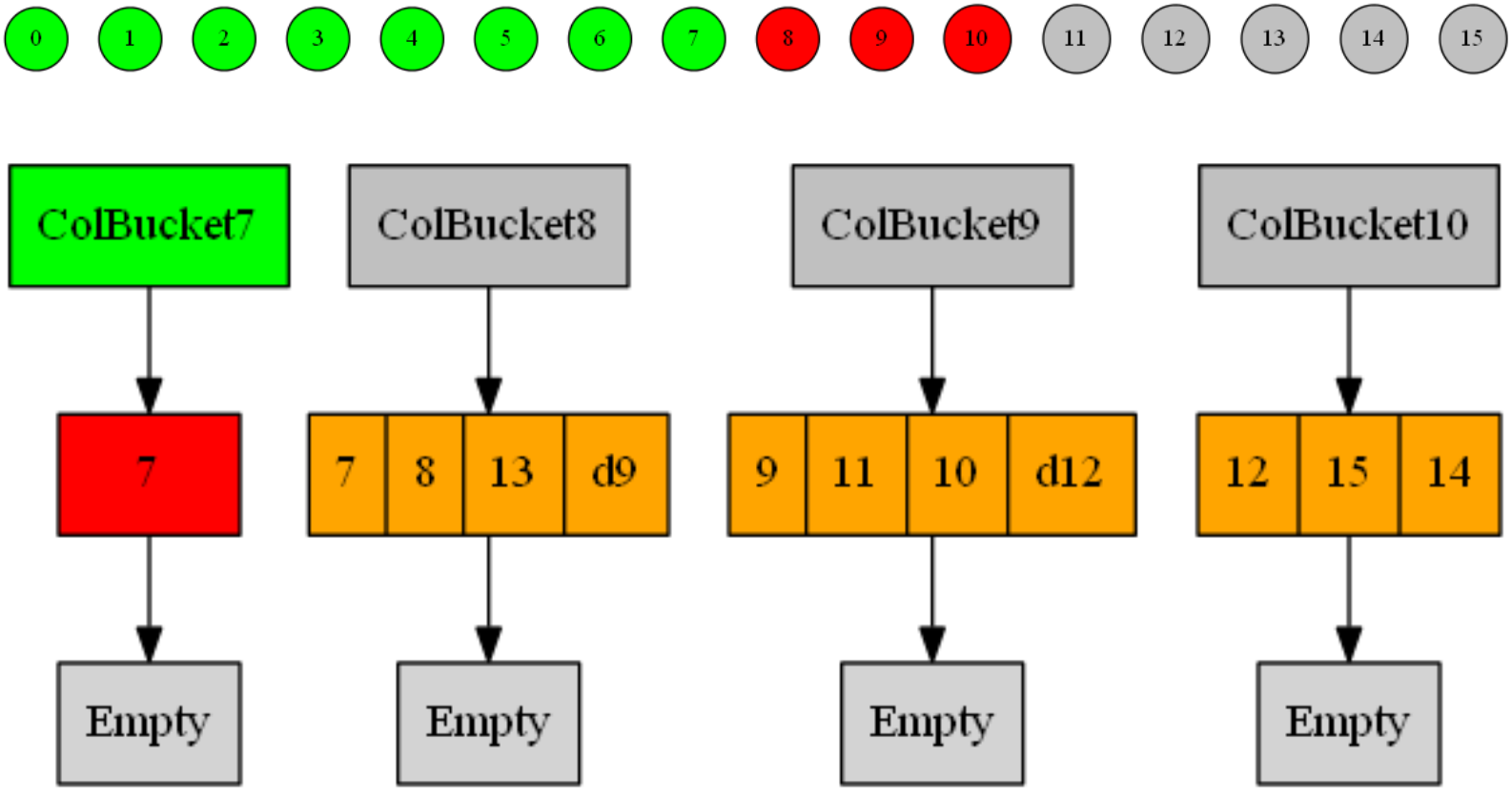

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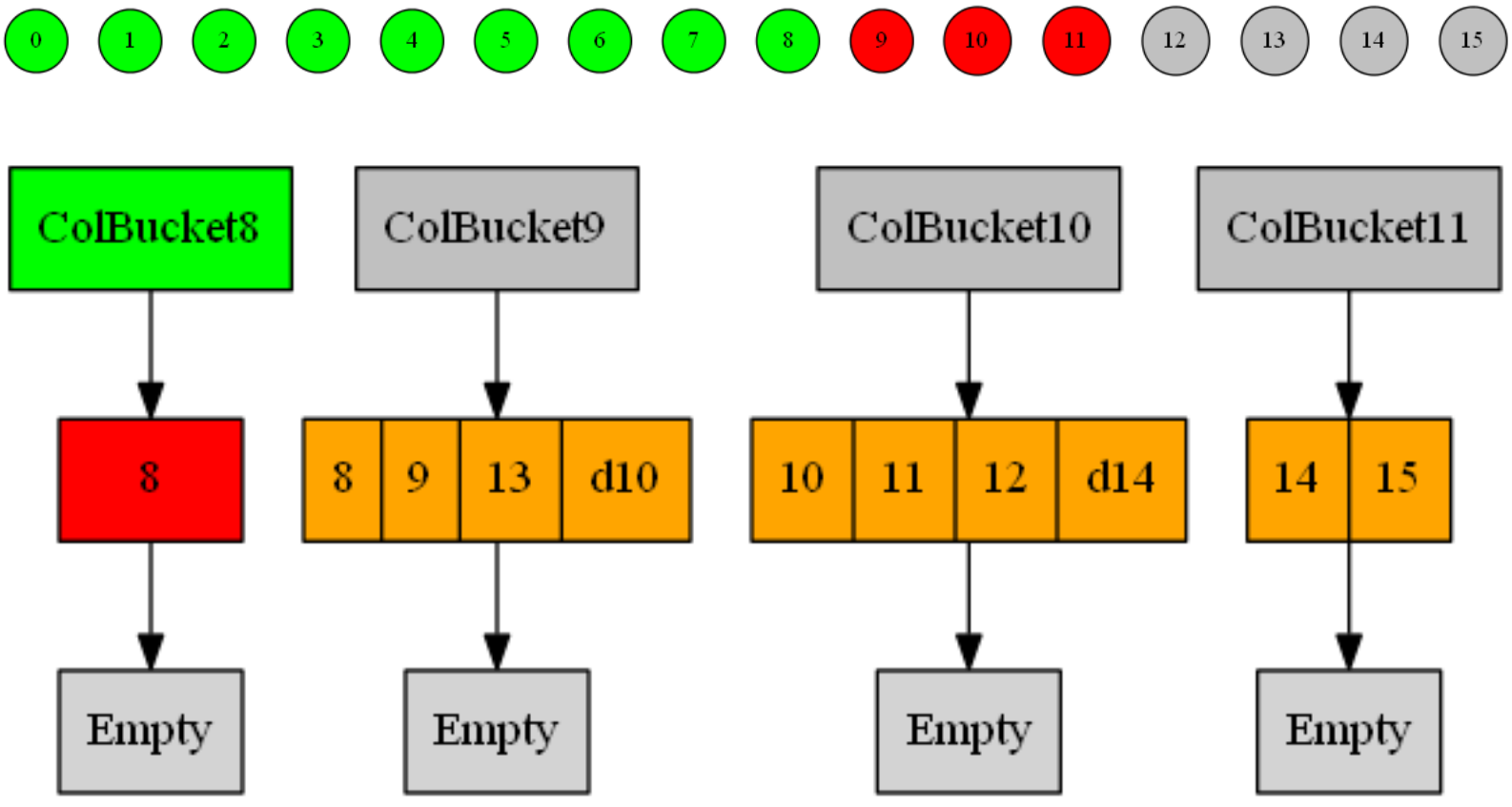

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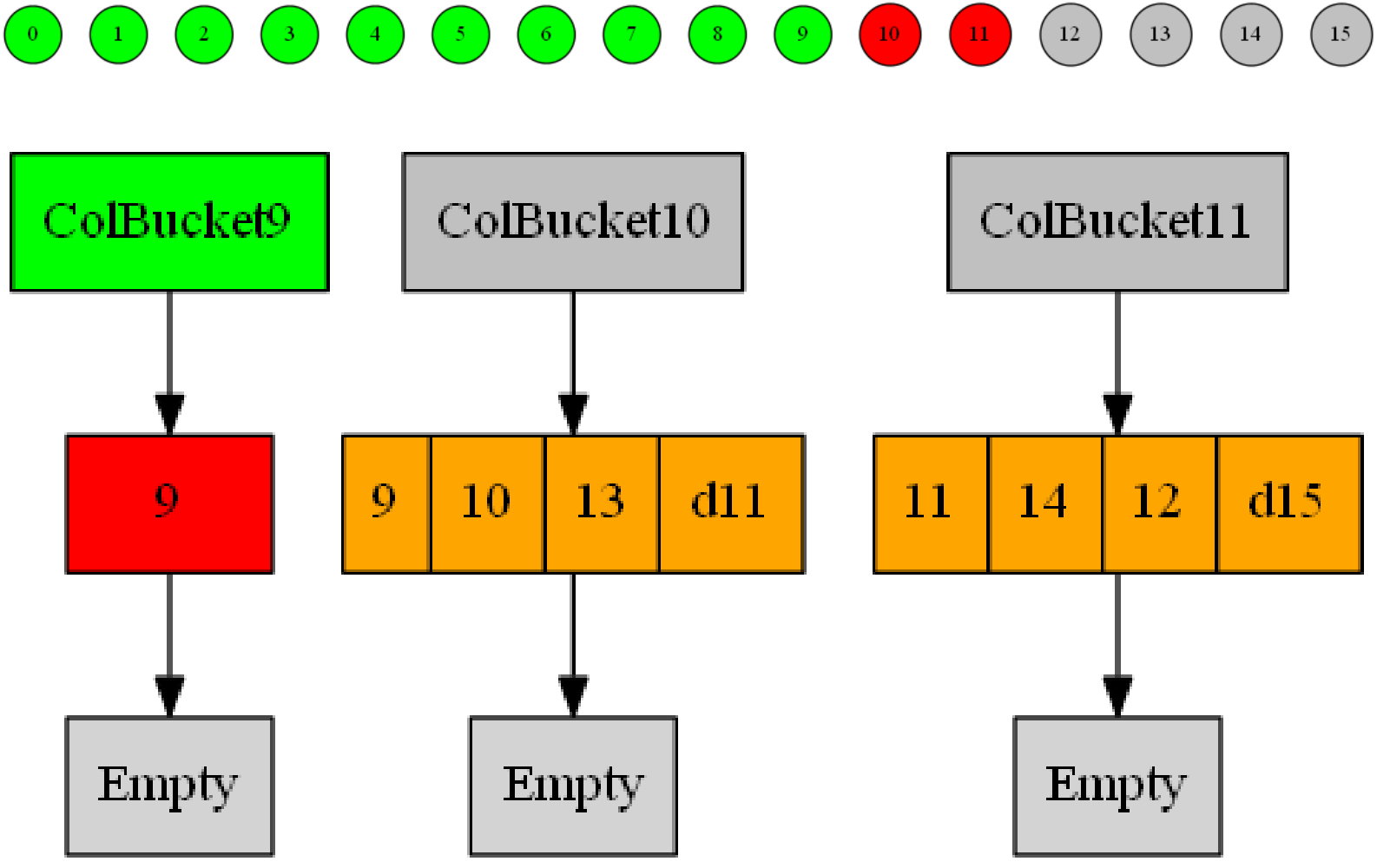

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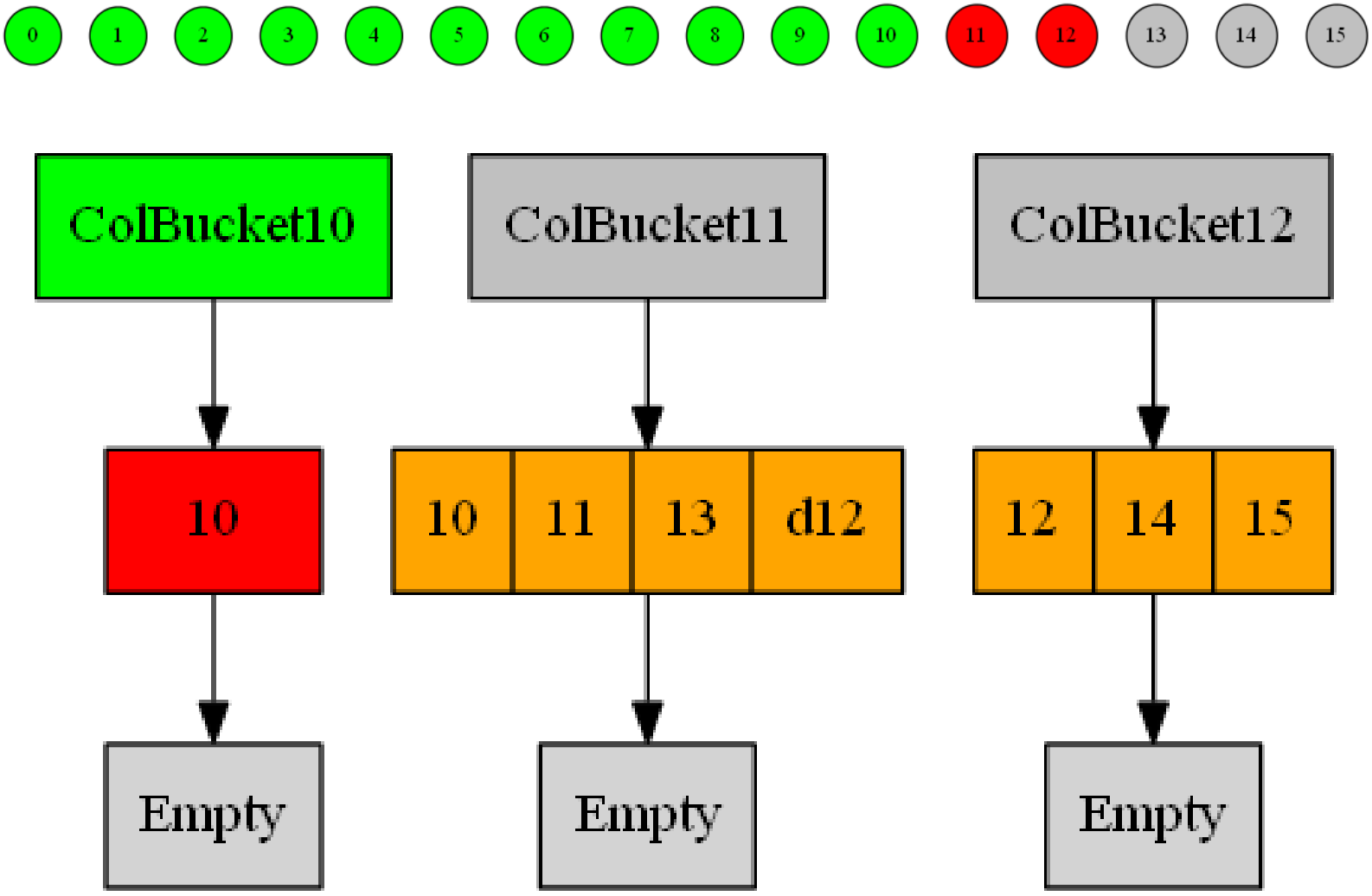

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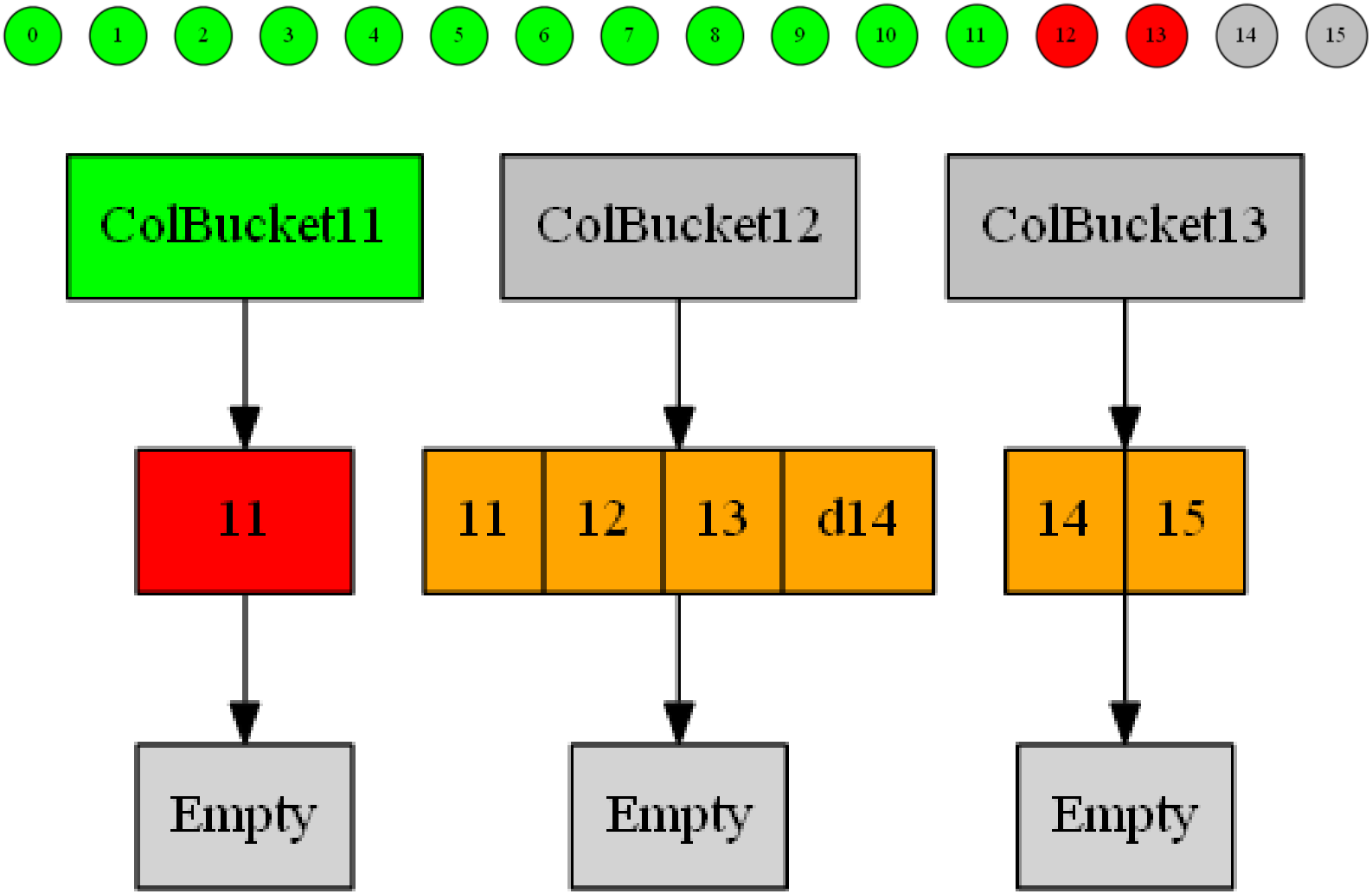

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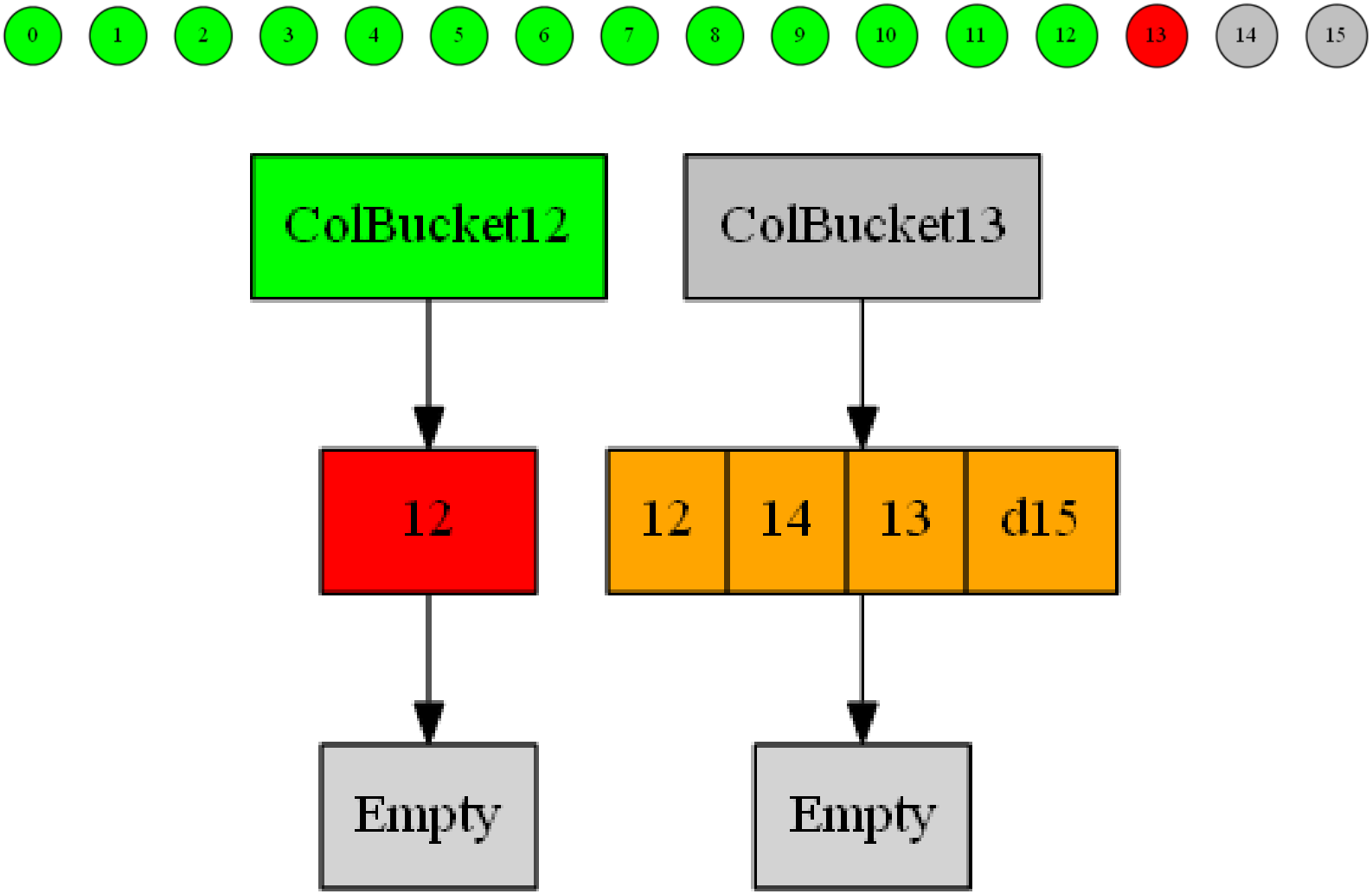

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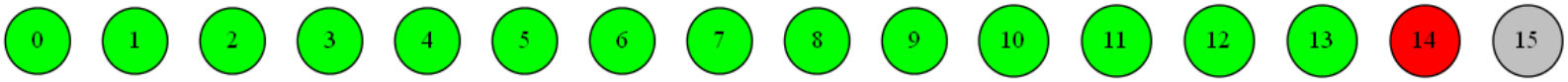

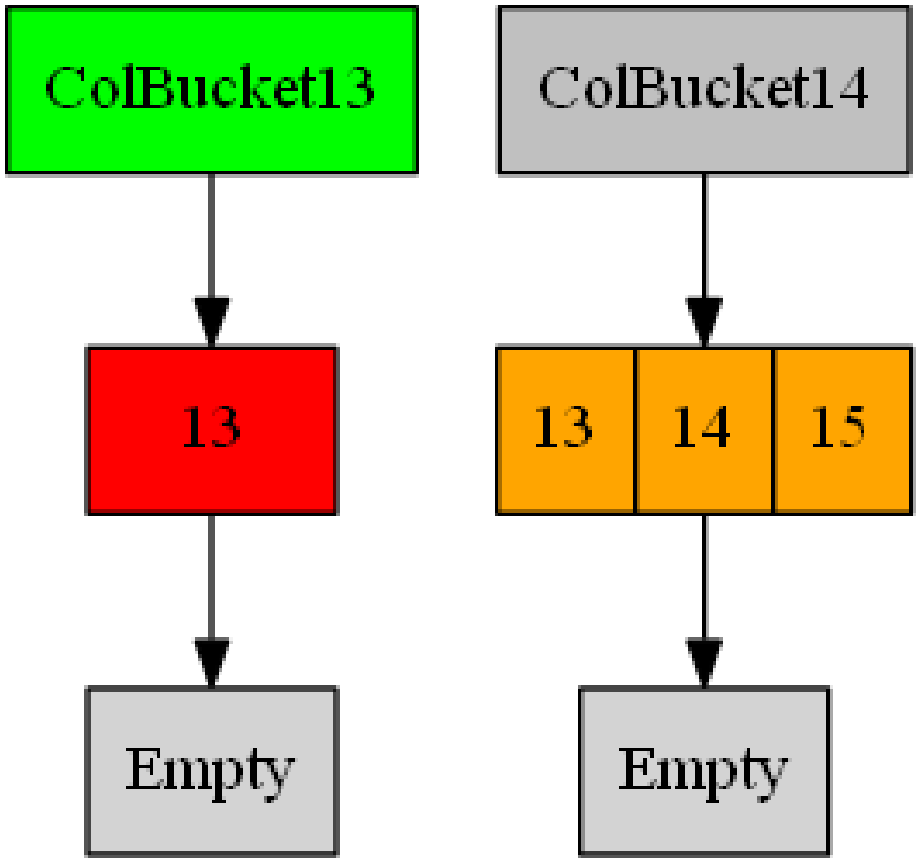

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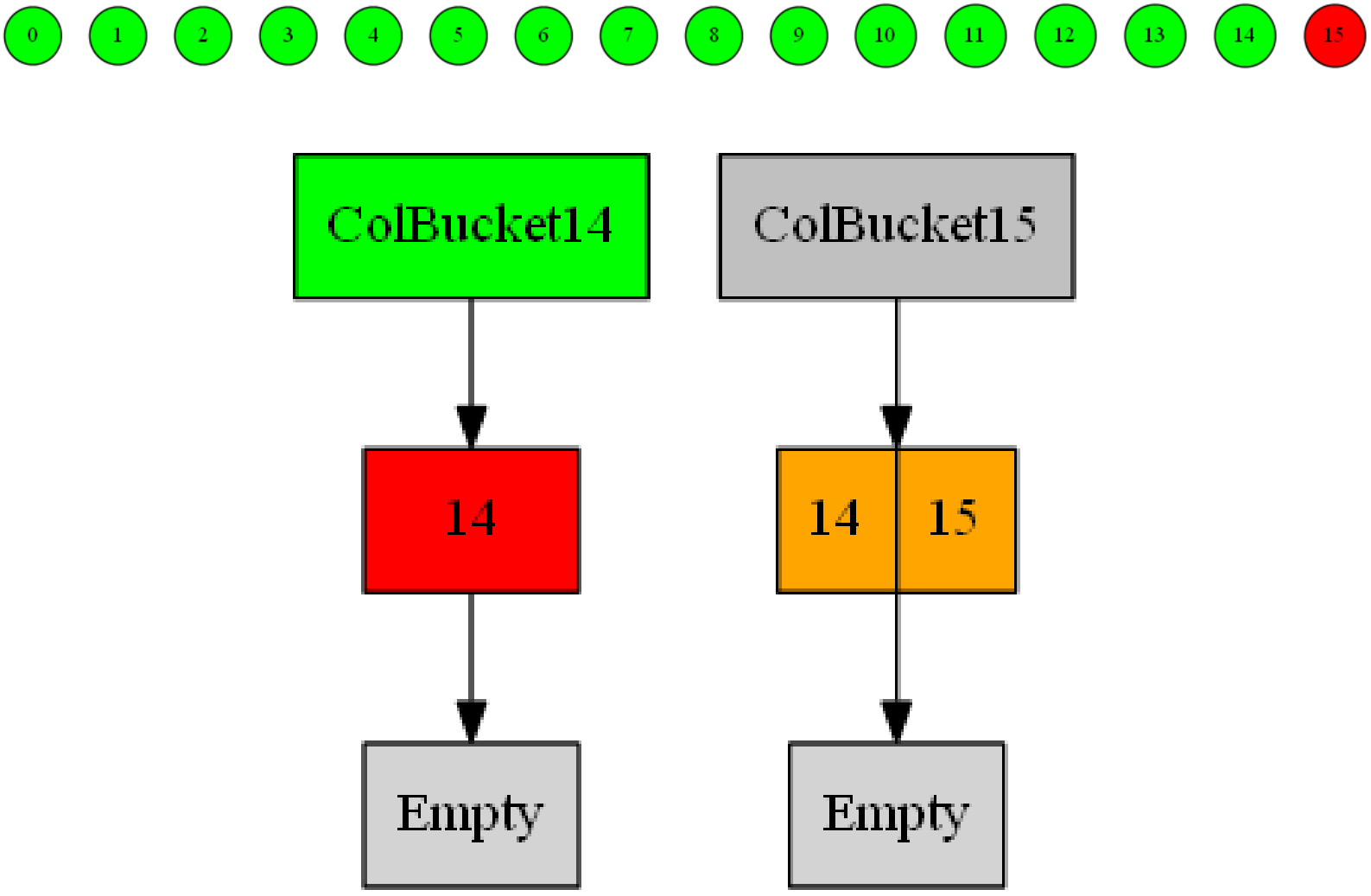

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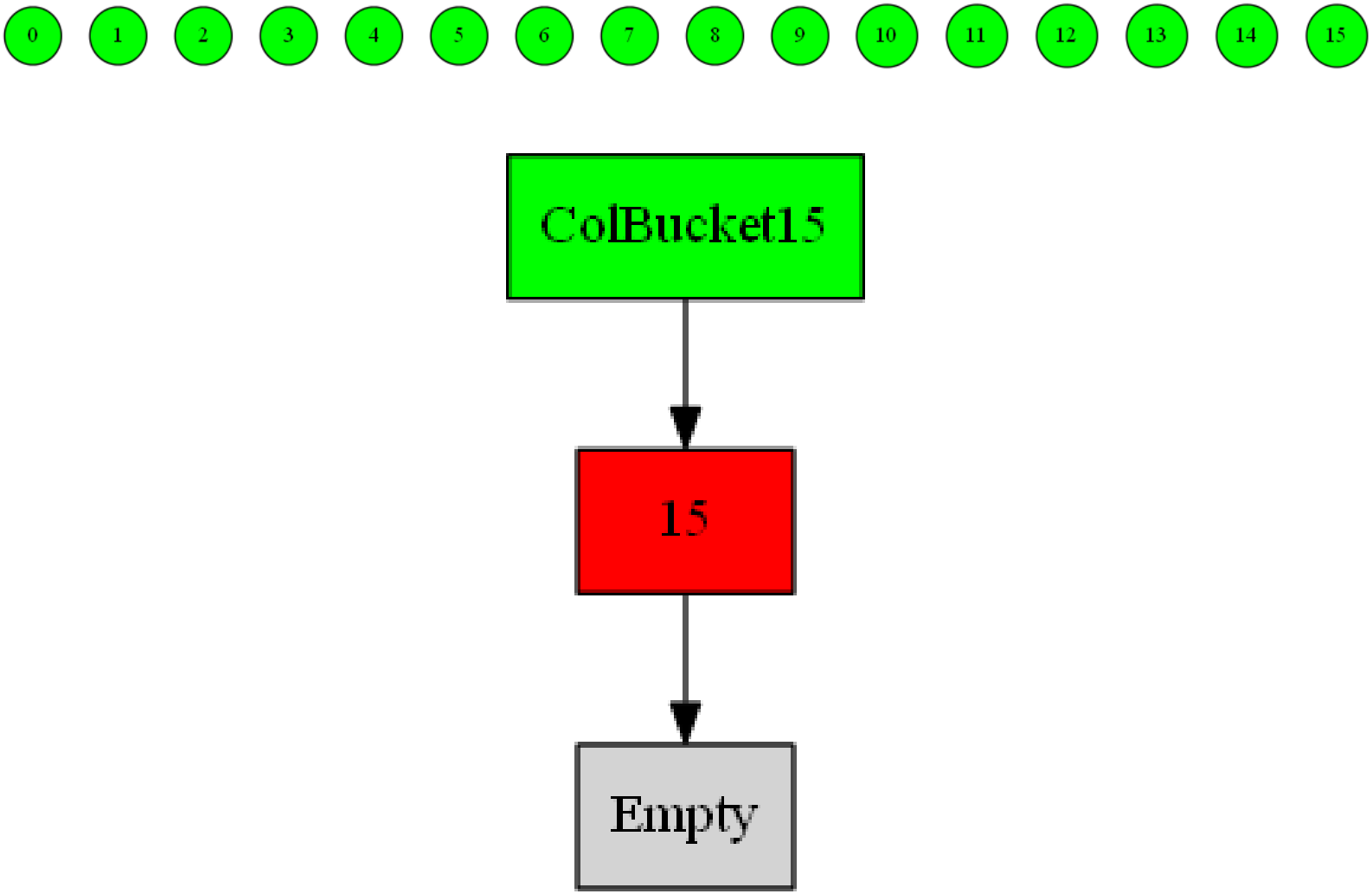

Future Work

- Add rank detection as a postprocessing step
- Consider bringing back Q in addition to R
- Decompose column elimination tree into subtrees to combat the GPU memory wall
- Subtree parallelism in a multi-GPU configuration
- Handle fronts that don't fit in GPU memory (MAGMA-style)
- Questions?